On Robustness of the Generalized Proportional Controller for Traffic Signal Control

Gustav Nilsson and Giacomo Como

Abstract— **We investigate robustness properties of the Generalized Proportional Allocation (GPA) policy that has been recently proposed for traffic signal control in urban networks. The GPA policy is fully decentralized, relies only on local information on the current congestion state, and requires no knowledge of the routing, the exogenous inflows, or the network structure. In previous work, we proved throughput optimality of the GPA policy, by showing that it is able to stabilize the traffic network dynamics whenever any controller is able to do so. In this paper, we first extend these results by showing that even when the measurements have offsets, the GPA policy maintains the same stability properties as with exact measurements. A comparison between the GPA and the MaxPressure traffic signal controllers with respect to robustness is also performed in a microscopic traffic simulator, where it is shown that while the GPA can handle offsets in the measurements, the MaxPressure controller may make vehicles wait forever.**

Index Terms— **Traffic Signal Control, Transportation Networks, Decentralized Control**

I. INTRODUCTION

With the recent development of new sensing technology for traffic measurements, such as cameras [1], together with more rapidly changing traffic demands due to, e.g., online routing, there is a growing need for feedback control in transportation networks. This to easily adopt new settings and keep the network operating smoothly under high demands.

While the earliest traffic signals were operated manually, better solutions to control traffic signals have been developed in line with the overall technological development. While several fairly complex solutions for control of traffic signals have been proposed, such as SCAT [2], SCOOT [3], and UTOPIA [4], those strategies lack formal guarantees of stability and are not decentralized.

One feedback based control strategy with provable stability properties for traffic signal control is the MaxPressure controller, presented in [5], [6]. It is an adaptation of the BackPressure controller [7], originally designed for queueing networks, to transportation networks. While the MaxPressure controller is decentralized and throughput optimal, the controller requires information about the routing, i.e., how vehicles turn in each junction. If the routing information is available to the controller, MPC-like solutions are possible as well, as proposed in e.g., [8], [9], [10], [11].

The Generalized Proportional Allocation (GPA) controller, recently proposed in [12], is decentralized and a throughput optimal controller for traffic signals. Throughput optimality means that no other controller can handle a larger amount of exogenous inflow compared to the GPA controller. Moreover, the GPA controller does not require any information about the routing. The ideas behind the GPA controller are related to another scheduler originally designed for communication networks, namely Proportional Fairness [13], [14].

In practical traffic applications, it is quite common that the sensors that measure the traffic volumes are not perfectly tuned, or the traffic volumes in the queues are not perfectly estimated from, e.g., loop detectors. It therefore makes sense to ensure that control strategies for traffic signals can perform well, even if the measurements are not perfect. In this paper, we investigate the robustness properties of the previously proposed GPA controller with respect to errors in the traffic volume measurements. Specifically, we will assume that the measurements of the traffic volumes are affected by a permanent additive disturbance. We will show that even if the GPA controller is not aware of that the measurements are affected by a disturbance, throughput-optimality and hence stability of the traffic network is still guaranteed.

The paper is organized as follows: the rest of this section is devoted to introducing some basic notation. In Section II, the dynamical model for the transportation network with signalized junctions is introduced. In Section III, we describe the GPA controller and show that despite the fact that the measurements are disturbed by an offset, the controller is still both stabilizing and being throughput optimal. In the following section, Section IV, we verify the GPA controller's performance in micro-simulator when there are measurement errors. We also investigate how the MaxPressure performs with the same faulty sensors. The paper is concluded with a few pointers to future work.

A. Notation

Let $\mathbb{R}_{(+)}$ denote the (non-negative) real numbers. For a finite set $\mathcal{A}$, $\mathbb{R}^{\mathcal{A}}$ denotes the set of all vectors indexed by the elements in $\mathcal{A}$. For a vector $a \in \mathbb{R}^n$, we let $\text{diag}\,(a) \in \mathbb{R}^{n \times n}$ be a matrix with the components of a on diagonal and all off-diagonal elements zero. With $\mathbb{1}$ we denote a vector whose all elements equals one. For a subset $\mathcal{S}$ of a metric space $\mathcal{M}$, $\text{int}(\mathcal{S})$ denotes the interior of the set $\mathcal{S}$ and for a $x \in \mathcal{M}$, $\text{dist}(x, \mathcal{S})$ denotes the shortest distance between x and $\mathcal{S}$.

G. Nilsson is with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA. gustav.nilsson@gatech.edu

G. Como is with the Department of Mathematical Sciences, Politecnico di Torino, Italy, and the Department of Automatic Control, Lund University, Sweden. giacomo.como@polito.it

This research was carried on within the framework of the MIUR-funded *Progetto di Eccellenza* of the *Dipartimento di Scienze Matematiche G.L. Lagrange*, CUP: E11G18000350001, and was partly supported by the *Compagnia di San Paolo* and the Swedish Research Council.

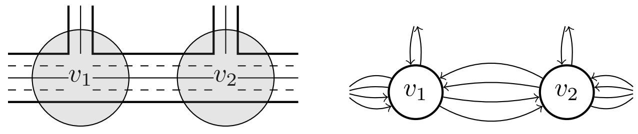

Fig. 1. Example of a part of a transportation network and the corresponding the graph representation. Each junction is a node, and each lane between the junctions is a link, which we refer to as a cell.

II. MODEL

As in [12], the topology of the transportation network is modeled as a multigraph with flow capacities $\mathcal{G} = (\mathcal{V}, \mathcal{E}, c)$ where $\mathcal{V}$ is the set of signalized junctions and $\mathcal{E}$ is the set of cells between the junctions where vehicles can propagate. In contrast to a graph, by using a multigraph we allow for several cells between two junctions, as illustrated in Figure 1. The traffic volumes in the cells will be the state space, and they will be denoted $x \in \mathcal{X}$ where $\mathcal{X} = \mathbb{R}_+^{\mathcal{E}}$. The outflow from each cell $i \in \mathcal{E}$ is limited by a fixed outflow capacity, denoted $c_i \in \mathbb{R}_+$ and the vector of all outflow capacities is denoted $c \in \mathbb{R}_+^{\mathcal{E}}$. Moreover, we introduce the outflow capacity matrix $C = \operatorname{diag}(c)$ and we will see later on why it is convenient.

To a subset of the cells, there is an exogenous inflow of vehicles. For a given cell $i \in \mathcal{E}$, the exogenous inflow is denoted $\lambda_i \in \mathbb{R}_+$ and the vector of all exogenous inflows is denoted $\lambda \in \mathbb{R}_+^{\mathcal{E}}$.

To model how the vehicles propagate through the network, we introduce the routing matrix $R \in \mathbb{R}_+^{\mathcal{E} \times \mathcal{E}}$. An entry R_{ij} represents the fraction of vehicles that will move from cell $i \in \mathcal{E}$ to cell $j \in \mathcal{E}$. Hence $R_{ij} = 0$ when cell i is not connected to cell j through a node. Moreover, for each cell $i \in \mathcal{E}$, the routing matrix has to satisfy $\sum_j R_{ij} \leq 1$, where $1 - \sum_j R_{ij}$ is the fraction of flow that will leave the network after flowing out from cell i.

A pair (λ, R) of an exogenous inflow vector and a routing matrix is said to be *in-connected* if for every $j \in \mathcal{E}$, there exists a cell $i \in \mathcal{E}$ with $\lambda_i > 0$ and a length $l \geq 0$ path $i = i_0, i_1, \ldots, i_l = j$ such that $\Pi_{1 \leq k \leq l} R_{i_{k-1}, i_k} > 0$. Moreover, a pair (λ, R) is said to be *out-connected* if for every cell $i \in \mathcal{E}$ with $\lambda_i > 0$ there exists a cell $j \in \mathcal{E}$ with $\sum_k R_{jk} < 1$ and a length $l \geq 0$ path $i = i_0, i_1, \ldots, i_l = j$ such that $\Pi_{1 \leq k \leq l} R_{i_{k-1}, i_k} > 0$. A routing matrix R is said to be out-connected if the pair (λ, R) is out-connected for every choice of λ.

To model the fact that a limited amount of cells can receive green light service simultaneously at a signalized junction, we introduce the set of phases. For a given junction $v \in \mathcal{V}$, let $\mathcal{E}^v$ denote the set of incoming cells to the junction. The set of phases then consists of subsets of the incoming cells $\mathcal{E}^v$. Formally, we define the set of phases $\mathcal{P}^v$ for a junction $v \in \mathcal{V}$ as $\mathcal{P}^v \subseteq \{\mathcal{Q} \mid \mathcal{Q} \subseteq \mathcal{E}^v\}$. The set of phases can be described through a binary matrix $P^{(v)} \in \{0,1\}^{\mathcal{E}^v \times \mathcal{P}^v}$, where each column corresponds to one phase,

$$P_{i,j}^{(v)} = \begin{cases} 1 & \text{if cell } i \in \mathcal{E}^v \text{ is activated in phase } j \in \mathcal{P}^v, \\ 0 & \text{if cell } i \in \mathcal{E}^v \text{ is not activated in phase } j \in \mathcal{P}^v. \end{cases}$$

From the phase matrices for each junction, the global phase matrix is defined as

$$P = \begin{bmatrix} P^{(1)} & & & \\ & P^{(2)} & & \\ & & \ddots & \\ & & & P^{(v)} \end{bmatrix}.$$

We will throughout the paper make the natural assumption that each cell belongs to at least one phase, i.e., $P\mathbb{1} \geq \mathbb{1}$. Moreover, we will say that the phases are *orthogonal* if each cell only belongs to one phase, i.e., $P\mathbb{1} = \mathbb{1}$. With the definition of phases, we can now provide a full definition of a flow network:

Definition 1 (Flow network): A flow network $\mathcal{N} = (\mathcal{G}, R, \lambda, P)$ is a tuple consisting of a multigraph with flow capacities $\mathcal{G}$, a routing matrix R, a vector of exogenous inflow λ, and a phase matrix P.

The dynamics of the flow network follows from the mass conservation law: the change of traffic volume is the difference between the combination of the exogenous and upstream inflows and the outflow from the cell, i.e.,

$$\dot{x} = \lambda - (I - R^T)z\,, \quad x \geq 0\,, \tag{1}$$

where $z \in \mathbb{R}_+^{\mathcal{E}}$ is the vector of outflows from the cells.

Apart from the outflow capcity, the outflow from each cell is limited by the phase activation. The task of the signalized junction controller is to control the fraction of time that each phase should be activated. To model this, we introduce the set of control signals as

$$\mathcal{U} = \Pi_{v \in \mathcal{V}} \mathcal{U}^v\,,$$

where

$$\mathcal{U}^v = \left\{ u \in \mathbb{R}_+^{\mathcal{P}^v} \mid \mathbb{1}^T u \leq 1 \right\},$$

where $1 - \sum_{q \in \mathcal{P}^v} u_q$ is the fraction of the cycle allocated to phase shifts. With the set of phases and control signals introduced, the outflow z in (1) is then limited by

$$0 \leq z \leq CPu\,, \quad u \in \mathcal{U}\,. \tag{2}$$

While the dynamics specified by (1) and (2) allow for several choices of the outflow vector z, we will further on make the assumption that if there are vehicles present in a cell, they will leave the cell with the maximum flow allowed by the controller, i.e.,

$$x^T(CPu - z) = 0\,. \tag{3}$$

Under this assumption, together with the assumption that R is out-connected, it has been showed that the dynamical system (1)–(3) has a unique solution both when the control action u is determined by an open loop controller [15] for piece-wise constant inflows λ, and when the control action is a Lipschitz continuous function of the state x [16], i.e., closed loop control, for time-varying inflows λ.

A flow-network with dynamics (1)–(3) is said to be *stable* if the traffic volumes $x(t)$ remain bounded in time t. This implies that with a continuous inflow of vehicles, the vehicles entering the network will eventually leave the network as well. Whether a traffic signal controller is able to stabilize the flow network or not, depends upon the magnitudes of the exogenous inflows λ. Since R is out-connected, it follows from e.g., [17, Theorem 2], that $(I - R^T)$ is invertible, and the arrival rates at equilibrium can then be computed as

$$a = (I - R^T)^{-1}\lambda\,.$$

A control strategy u is said to be throughput optimal if the following definition is satisfied:

Definition 2 (Throughput-optimal): A control strategy u for a flow network $\mathcal{N} = (\mathcal{G}, R, \lambda, P)$ with an out-connected routing matrix R is said to be throughput optimal if for every $\lambda \in \mathbb{R}_+^{\mathcal{E}}$ such that

$$(I - R^T)^{-1}\lambda \in \text{int}(\mathcal{Z})\,, \tag{4}$$

where

$$\mathcal{Z} = \{z \in \mathbb{R}_+^{\mathcal{E}} \mid 0 \leq z \leq CP\nu \text{ for some } \nu \in \mathcal{U}\}\,,$$

is able to stabilize the flow network with dynamics given by (1)–(3).

The reason for including the constraint (4) in the definition, is that it has already been shown in [12] that if λ does not fulfill condition (4) it is impossible for any traffic signal controller to stabilize the flow network.

While the definition of throughput-optimality holds for any structure of traffic signal controllers, we will for the rest of this paper focus on feedback based controllers $\upsilon(x)$, i.e., the control signal u depends on the traffic volumes x such that $u = \upsilon(x)$.

III. GPA Control with Offsets in the Measurements

In this section we will show that despite the measurements for the GPA controller is affected by an offset, the stability properties still hold. To emphasize the distributed nature of the controller, for each junction $v \in \mathcal{V}$, we let $x^{(v)}$ denote the projection of x on $\mathcal{E}^v$. Under the assumption that the phases are orthogonal, the standard GPA controller, $\upsilon_q^{(v)}(x)$, that computes the fraction of time that each phase $q \in \mathcal{P}^v$ for each junction $v \in \mathcal{V}$ based on the traffic volumes x, for each phase $q \in \mathcal{P}^v$, is given by [12]:

$$\upsilon_q^{(v)}(x^{(v)}) = \frac{(P^Tx)_q}{\kappa_v + \sum_{r\in\mathcal{P}^v}(P^Tx)_r} = \frac{\sum_i P_{iq}x_i}{\kappa_v + \sum_{j\in\mathcal{E}^v} x_j}\,. \tag{5}$$

In the expression above, $\kappa_v > 0$ is modeling parameter introduced to capture the fact that a fraction of the green light service cycle has to be devoted to phase shifts. The outflow z_i from each cell $i \in \mathcal{E}$ is then limited by

$$\zeta_i(\tilde{x}) = CP\upsilon(x)\,. \tag{6}$$

While the analysis in [12] has been done under the assumption that the measurements are perfect, we will in this paper assume that the measurements are affected by an offset. Let $\tilde{x}$ denote the measured traffic volumes, such that $\tilde{x}_i = x_i + d_i$ where $d_i \geq 0$ is a non-negative constant offset in the measurement. Observe that this offset is unknown to the controller, and only introduced in the analysis to be able to compare the measured values with the true traffic volumes. Under the assumption that the phases are orthogonal, for each junction $v \in \mathcal{V}$ the phase activation $\upsilon_q^{(v)}$ of phase $\upsilon_q^{(v)}$ is now determined by

$$\begin{aligned}\upsilon_q^{(v)}(\tilde{x}^{(v)}) &= \frac{(P^T\tilde{x})_q}{\kappa_v + \sum_{r\in\mathcal{P}^v}(P^T\tilde{x})_r} \\ &= \frac{\sum_i P_{iq}(x_i + d_i)}{\kappa_v + \sum_{j\in\mathcal{E}^v}(x_j + d_i)}\,.\end{aligned} \tag{7}$$

Remark 1: With positive offsets $d_i > 0$, the controlled outflow $\zeta_i(x)$ is strictly positive for an empty cell x_i. If the inflow to the cell is less than $\zeta_i(x)$, it must hold that $z < \zeta_i(x)$. This justifies the need for a differential inclusion description of the dynamics. In previous work [18], it has been shown that if each phase only consists of one single cell the differential inclusion can be avoided. This since it holds that $\lim_{x_i\to 0^+}\zeta_i(x) = 0$. This property is not even true for this simplified phase structure when the measurements have offsets.

The following theorem ensures both stability, i.e., that the traffic volumes will stay bounded, and throughput optimality of the GPA controller, when the traffic volume measurements are affected by a constant offset:

Theorem 1: Consider the flow network $\mathcal{N} = (\mathcal{G}, R, \lambda, P)$ with the dynamics given by (1)–(3), orthogonal phases, and the controller u given by (7). Assume that (λ, R) is both in- and out-connected and that measurements of traffic volumes are disturbed by a constant offset $d \in \mathbb{R}_+^{\mathcal{E}}$. Introduce the set

$$\mathcal{X}^* = \{x \in \mathcal{X} \mid \zeta(x) \geq 0, x^T(\zeta(x) - a) \geq 0\}\,.$$

If $(I - R^T)^{-1}\lambda \in \text{int}(\mathcal{Z})$, for any initial traffic volumes $x(0) \in \mathcal{X}$, the traffic volumes $x(t) \to \mathcal{X}^*$, as $t \to +\infty$.

Proof: The proof is a LaSalle-like argument. We start by introducing the function $H : \mathbb{R}_+^{\mathcal{E}} \times \mathbb{R}_+^{\mathcal{P}} \to \mathbb{R}$ as

$$H(x, \nu) = \sum_{i\in\mathcal{E}}(x_i + d_i)\log\frac{(CP\nu)_i}{a_i} + \sum_{v\in\mathcal{V}}\kappa_v \log\frac{1 - \mathbb{1}^T\nu^{(v)}}{b_v}\,, \tag{8}$$

where

$$b_v = 1 - \min_{\substack{\nu\,\in\,\mathcal{U}^v\,:\\ C^{(v)}P^{(v)}\nu\,\geq\, a^{(v)}}} \mathbb{1}^T\nu\,.$$

Observe that $b_v > 0$, due to the assumption that $a \in \text{int}(\mathcal{Z})$. Let

$$V(x) = \max_{v\in\mathcal{U}} H(x, v)\,. \tag{9}$$

We will now show that $V(x(t))$ is non-increasing with respect to time t and strictly decreasing outside the set $\mathcal{X}^*$. We start by observing the following properties of $V(x)$:

i) The controller υ in (7) with offsets in the measurements is the maximizer in (9), i.e.,

$$V(x) = H(x, \upsilon(\tilde{x}))\,.$$

ii) For all $x \in \mathcal{X}$, it holds that $V(x) \geq 0$.
iii) For all $x_i > 0$, it holds that

$$\frac{\partial V}{\partial x_i} = \omega_i(x) \,, \quad \text{where} \quad \omega_i(x) = \log\left(\frac{\zeta_i(x)}{x_i}\right) .$$

To show property i), introduce the function $L : \mathbb{R}_+^{\mathcal{E}} \times \mathbb{R}_+^{\mathcal{P}} \times \mathbb{R}_+^{\mathcal{V}} \to \mathbb{R}$ as $L(x, \nu, \gamma) = H(x, \nu) + \sum_{k \in \mathcal{V}} \gamma_k (1 - \mathbb{1}^T \nu^{(k)})$. Necessary conditions for optimum are that

$$\frac{\partial L}{\partial \nu_q^{(v)}} = \frac{1}{\nu_q^{(v)}} \sum_{i \in \mathcal{E}^v} P_{iq}^{(v)} (x_i + d_i) - \frac{1}{1 - \mathbb{1}\nu^{(v)}} \kappa_v - \gamma_v = 0 \,,$$

for all $q \in \mathcal{P}^v$ and all $v \in \mathcal{V}$. From the complementary slackness principle, we get that either $1 - \mathbb{1}^T \nu^{(k)}$ is zero, which clearly can not be a maximum, or $\gamma_k = 0$. For the latter case, it holds that

$$\frac{1}{\kappa_v} \sum_{i \in \mathcal{E}^v} P_{iq}^{(v)} (x_i + d_i) = \frac{\nu_q^{(v)}}{1 - \mathbb{1}^T \nu^{(v)}} \,, \tag{10}$$

for all $q \in \mathcal{P}^v$ and all $v \in \mathcal{V}$. Summing up the expression above over all phases $q \in \mathcal{P}^v$ and using the fact that the phases are orthogonal yields

$$\frac{1}{\kappa_v} \sum_{i \in \mathcal{E}^v} (x_i + d_i) = \frac{\mathbb{1}^T \nu^{(v)}}{1 - \mathbb{1}^T \nu^{(v)}} \,,$$

and hence

$$\mathbb{1}^T \nu^{(v)} = \frac{\sum_{i \in \mathcal{E}^v} (x_i + d_i)}{\kappa_v + \sum_{i \in \mathcal{E}^v} (x_i + d_i)} \,. \tag{11}$$

By combining (10) and (11) we get

$$\upsilon_q^{(v)} = \frac{\sum_{i \in \mathcal{E}^v} P_{iq} (x_i + d_i)}{\kappa_v + \sum_{i \in \mathcal{E}^v} (x_i + d_i)} \,,$$

which, together with the concavity of (8), proves that (7) is the maximizer in (9).

Property ii) follows from the fact that if we pick a $\tilde{\nu} \in \mathcal{U}$ such that $(CP\tilde{\nu})_i \geq a_i$, for all $i \in \mathcal{E}$ and $1 - \mathbb{1}^T \tilde{\nu}^{(k)} = b_k$, which is possible to find due to the definition of b_k, then

$$V(x) = \max_{\nu \in \mathcal{U}} H(x, \nu) \geq H(x, \tilde{\nu}) \geq 0 \,.$$

To show property iii): let $x^\epsilon \in \mathcal{X}$ be a vector such that $x_i^\epsilon = x_i + \epsilon$ and x_j^ϵ for all $j \neq i \in \mathcal{E}$. Then

$$\begin{aligned}
&V(x^\epsilon) - V(x) = \\
&\sum_{j \in \mathcal{E}} (x_j^\epsilon + d_j) \log \frac{\zeta_j(x^\epsilon)}{a_j} + \sum_{v \in \mathcal{V}} \kappa_v \log \frac{1 - \mathbb{1}^T \upsilon^{(v)}(x^\epsilon)}{b_v} \\
&- \sum_{j \in \mathcal{E}} (x_j + d_j) \log \frac{\zeta_j(x)}{a_j} + \sum_{v \in \mathcal{V}} \kappa_v \log \frac{1 - \mathbb{1}^T \upsilon^{(v)}(x)}{b_v} \\
&\geq \sum_{j \in \mathcal{E}} (x_j^\epsilon + d_j) \log \frac{\zeta_j(x)}{a_j} + \sum_{v \in \mathcal{V}} \kappa_v \log \frac{1 - \mathbb{1}^T \upsilon^{(v)}(x)}{b_k} \\
&- \sum_{j \in \mathcal{E}} (x_j + d_j) \log \frac{\zeta_j(x)}{a_j} + \sum_{v \in \mathcal{V}} \kappa_v \log \frac{1 - \mathbb{1}^T \upsilon^{(v)}(x)}{b_k} \\
&\qquad\qquad\qquad\qquad\qquad\qquad = \epsilon \log \frac{\zeta_i(x)}{a_i} \,,
\end{aligned}$$

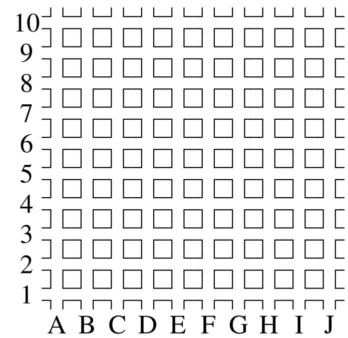

Fig. 2. The Manhattan-like grid used in the simulations.

where the inequality follows from the fact that $H(x^\epsilon, \upsilon(x^\epsilon)) = \max_{\nu \in \mathcal{U}} H(x^\epsilon, \nu) \geq H(x^\epsilon, \upsilon(x))$. In the same manner, we have that

$$V(x^\epsilon) - V(x) \leq \epsilon \log \frac{\zeta_i(x^\epsilon)}{a_i} \,.$$

By combining the bound we get

$$\log \frac{\zeta_i(x)}{a_i} \leq \frac{1}{\epsilon} (V(x^\epsilon) - V(x)) \leq \log \frac{\zeta_i(x^\epsilon)}{a_i} \,.$$

Since $\zeta(x)$ depends continuously on x, letting $\epsilon \to 0$ proves the statement in iii).

The rest of the proof for that $V(x(t))$ is non-increasing with respect to time t and strictly decreasing outside the set $\mathcal{X}^*$ follows the same way as the proof of Theorem 1 in [19]. ■

Remark 2: Observe that the GPA controller is still throughput optimal, despite the offsets in the traffic volume measurements. This implies that even if the sensors are not perfectly tuned, the controller is still able to handle the same amount of inflow that any other stabilizing controller is able to handle with perfect measurements.

Remark 3: Even though the offsets in the measurements can be caused by badly tuned sensors, one can also deliberately introduce offsets to change the equilibrium of the system. For instance, if the arrival rates at equilibrium are known and the equilibrium is unique, by adjusting the offsets one can get a controller that keeps the equilibrium close zero.

IV. Validations on a microscopic traffic simulator

In this section, we will investigate how the offsets in the measurements affect the GPA controller's performance in a microscopic simulator for vehicle traffic, SUMO [20]. The setting we are using is an artificial Manhattan-like grid, shown in Figure 2. All streets with an odd number or indexed by letter A, C, E, G, or I consist of one lane in each direction and the others consist of two lanes. This means that there are four different types of junctions in the network and the four types together with their phases are shown in Figure 3. The distances between the junctions, all signalized, are all 300 meters. Fifty meters before each junction, each street has an additional lane for vehicles turning left.

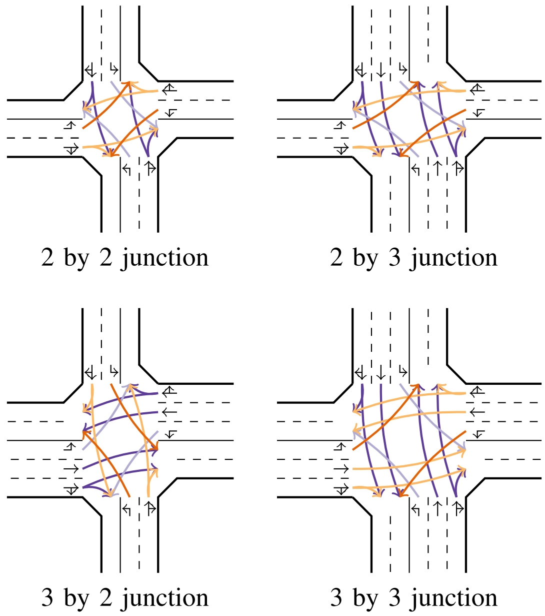

Fig. 3. The four different types of junctions present in the Manhattan grid, together with theirs set of phases.

For the simulations, we generate three different demand scenarios. All demands are generated for a fixed routing matrix, where in each junction 20 percent of the vehicles will turn left, 20 percent will turn right, and the rest will continue straight when they approach the junctions. All trips are started from the roads at the boundary of the network and end when the vehicle eventually leaves the network. For each road at the boundary and every second a vehicle will depart with probability $l = 0.05, 0.10, 0.15$. Departures are allowed for the first 3600 seconds, and the simulation will go on until the network is empty. This demand profile will cause different loads in different parts of the network; the central part will be more loaded than the boundary. It is worth emphasizing that the demand generation is done before the simulations, so all simulations with the same departure probability l will have to serve exactly the same demand profile. For a complete description about the scenario, see [21], where the same scenario is used without offsets in the measurments.

All the signalized junctions are equipped with sensors that are able to measure the number of vehicles queueing up, up to 50 meters before the junctions. The simulations are performed both when the sensors' measurements are exact and when they are affected by offsets. The offsets are set to be one vehicle for sensors located north and east of the junctions and two vehicles for sensors to the west of the junction. The sensors south of each junction are set to measure the queue lengths accurately.

A. GPA Controller

In the scenario, the phases are orthogonal and we will follow the discretization of the GPA controller presented in [21], which means that at the beginning of each cycle, the signal controller in each junction $v \in \mathcal{V}$ computes the fraction of the upcoming cycle that each phase $q \in \mathcal{P}^v$ should be activated according to (7). Since $1 - \sum_{q\in\mathcal{P}^v} \nu_q^{(v)}$ corresponds to the time for phase shifts, and this is in practice a fixed amount of time, a direct expression for the total cycle length can be obtained. The cycle length for the upcoming cycle is given by

$$T^{(v)}_{\text{cyc}}(t) = T_w n(t) + \frac{T_w n(t)}{\kappa_v} \sum_{i\in\mathcal{E}^v} \tilde{x}_i(t)\,,$$

where $n(t)$ is the number of phases that are going to be activated during the upcoming cycle, and $T_w > 0$ the time each clearance phase, i.e., when the traffic signal is showing a yellow light, should be activated. In the simulations we let $T_w = 4$ seconds.

B. MaxPressure Controller

To show that it is not certain that all throughput optimal controllers are able to handle offsets in the measurements, we will simulate the MaxPressure (MP) controller in the same simulation setting. In difference to the GPA controller, the MaxPressure controller needs information about the routing matrix R. For the simulations, we will assume that the MaxPressure controller has perfect knowledge of the routing matrix. With the knowledge of the routing matrix and under the assumption that the flow rates are the same for all phases, the MaxPressure controller works as follows: First, it computes the pressure, w_q, for each phase $q \in \mathcal{P}^v$ as

$$w_q(t) = \sum_{i\in q}\left(\tilde{x}_i(t) - \sum_{j\in\mathcal{E}} R_{ij}\tilde{x}_j(t)\right).$$

Then, the MaxPressure controller activates any phase in the set $\operatorname{argmax}_i w_i$. In the simulation we activate this phase for 10 seconds, then activate a clearance phase for 4 seconds. After that, the pressures are computed again and a new phase is activated.

C. Simulation Results

In Figure 4 simulations for the three different load scenarios are shown for both controllers, both when the sensors measure the actual queue lengths and when the measurements have offsets. For the GPA controller, the tuning parameter κ is set equal 5 for the scenarios with $l = 0.05$ and $l = 0.10$, since this has been shown to be a reasonable choice in [21] when the measurements are accurate. For the scenario with the highest load, κ was et equal to 15, since a lower κ will cause longer cycles and too large back-spill effects. The back-spills will then cause a grid-lock. This issue arises because of the finite vehicle storage in the lanes, and hence does not contradict the analysis in the previous sections. In the figure it can also be seen that although the MaxPressure controller performs well during the loading phase of the simulations, it is not able to drain the network when there are no vehicles flowing in anymore.

In Table I we show the running time until all the vehicles have left the network when the measurements have offsets. Interestingly, with the MaxPressure controller, this may never happen. This is due to the fact that if the measurements have

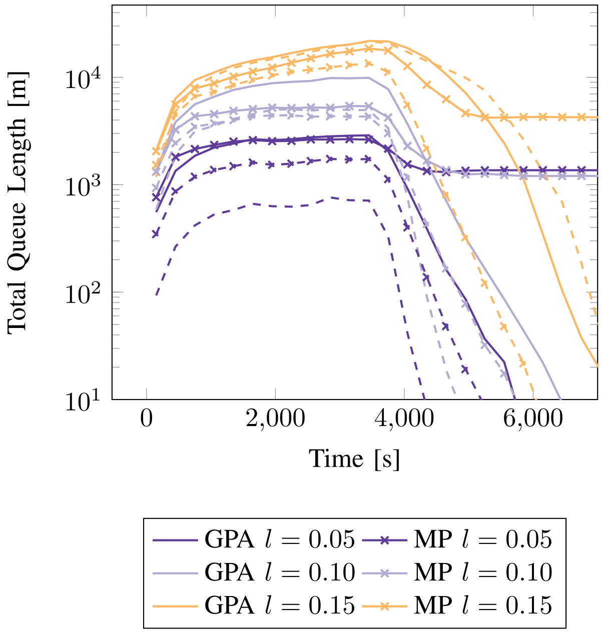

Fig. 4. The total queue length, i.e., the sum over all queue lengths for the 100 signalized junctions where each junction has between 8 and 12 incoming lanes, over time in the Manhattan grid with the GPA controller and the MaxPressure controller for different loads. Solid lines are with offsets in the measurements, dashed lines with true measurements. To improve readability, queue lengths are averaged over 300 seconds intervals.

TABLE I
TOTAL RUNNING TIME

Setting	Time to Empty [s]
MP $l = 0.05$	$+\infty$
MP $l = 0.10$	$+\infty$
MP $l = 0.15$	$+\infty$
GPA $l = 0.05$ $\kappa = 5$	6169
GPA $l = 0.10$ $\kappa = 5$	6600
GPA $l = 0.15$ $\kappa = 15$	7898

offsets, a queue with zero vehicles and positive offset may give its corresponding phase higher pressure compared to a queue with zero offset and a few vehicles. The MaxPressure controller will then repeatedly activate this phase with the empty queue and let the queueing vehicles wait forever.

V. CONCLUSIONS

We have studied robustness properties of the GPA controller, in the case when the traffic volume measurements are affected by offsets. We have shown that the GPA controller is still able to stabilize the network, without having any information about the offsets. Despite the offsets in the measurements, the GPA controller is still throughput optimal. Although the formal analysis is carried out in an averaged model, we have also shown in a microscopic traffic simulator that the GPA controller can handle this kind of disturbances well. We also observed that MaxPressure controller is more sensitive than the GPA controller to measurement offsets. In the future, we plan to investigate time-varying disturbances, both in the measurements and in the exogenous inflows, as well as dynamic routing (c.f. [22] and references therein).

REFERENCES

[1] A. Albiol, A. Albiol, and J. M. Mossi, "Video-based traffic queue length estimation," in *2011 IEEE International Conference on Computer Vision Workshops (ICCV Workshops)*, Nov 2011, pp. 1928–1932.

[2] A. G. Sims and K. W. Dobinson, "The Sydney coordinated adaptive traffic (SCAT) system philosophy and benefits," *IEEE Transactions on vehicular technology*, vol. 29, no. 2, pp. 130–137, 1980.

[3] D. I. Robertson and R. D. Bretherton, "Optimizing networks of traffic signals in real time-the SCOOT method," *IEEE Transactions on vehicular technology*, vol. 40, no. 1, pp. 11–15, 1991.

[4] V. Mauro and C. Di Taranto, "Utopia," in *Control, computers, communications in transportation*. Elsevier, 1990, pp. 245–252.

[5] P. Varaiya, *The Max-Pressure Controller for Arbitrary Networks of Signalized Intersections*. New York, NY: Springer New York, 2013, pp. 27–66.

[6] ——, "Max pressure control of a network of signalized intersections," *Transportation Research Part C: Emerging Technologies*, vol. 36, pp. 177 – 195, 2013.

[7] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1936–1948, Dec 1992.

[8] P. Grandinetti, C. C. de Wit, and F. Garin, "Distributed optimal traffic lights design for large-scale urban networks," *IEEE Transactions on Control Systems Technology*, pp. 1–14, 2018.

[9] G. Bianchin and F. Pasqualetti, "A network optimization framework for the analysis and control of traffic dynamics and intersection signaling," in *57th IEEE Conference on Decision and Control*, Dec 2018, pp. 1017–1022.

[10] Z. Hao, R. Boel, and Z. Li, "Model based urban traffic control, Part I: Local model and local model predictive controllers," *Transportation Research Part C: Emerging Technologies*, vol. 97, pp. 61 – 81, 2018.

[11] ——, "Model based urban traffic control, Part II: Coordinated model predictive controllers," *Transportation Research Part C: Emerging Technologies*, vol. 97, pp. 23 – 44, 2018.

[12] G. Nilsson and G. Como, "On generalized proportional allocation policies for traffic signal control," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 9643 – 9648, 2017, 20th IFAC World Congress.

[13] L. Massoulié, "Structural properties of proportional fairness: Stability and insensitivity," *The Annals of Applied Probability*, vol. 17, no. 3, pp. 809–839, 06 2007.

[14] N. S. Walton, "Concave switching in single and multihop networks," in *The 2014 ACM International Conference on Measurement and Modeling of Computer Systems*, ser. SIGMETRICS '14. New York, NY, USA: ACM, 2014, pp. 139–151.

[15] P. Hosseini and K. Savla, "Queue length simulation for signalized arterial networks and steady state computation under fixed time control," *ArXiv e-prints*, 2017. [Online]. Available: https://arxiv.org/abs/1705.07493v2

[16] G. Como and G. Nilsson, "On the well-posedness of dynamical flow networks with feedback-controlled outflows," 2020, under review. [Online]. Available: https://arxiv.org/abs/2001.06097

[17] G. Como and F. Fagnani, "From local averaging to emergent global behaviors: The fundamental role of network interconnections," *Systems & Control Letters*, vol. 95, pp. 70 – 76, 2016.

[18] G. Nilsson, P. Hosseini, G. Como, and K. Savla, "Entropy-like Lyapunov functions for the stability analysis of adaptive traffic signal controls," in *2015 54th IEEE Conference on Decision and Control (CDC)*, Dec 2015, pp. 2193–2198.

[19] G. Nilsson and G. Como, "Generalized proportional allocation policies for robust control of dynamical flow networks," 2019, under review. [Online]. Available: https://arxiv.org/abs/1907.02045v1

[20] D. Krajzewicz, J. Erdmann, M. Behrisch, and L. Bieker, "Recent development and applications of SUMO - Simulation of Urban MObility," *International Journal On Advances in Systems and Measurements*, vol. 5, no. 3&4, pp. 128–138, December 2012.

[21] G. Nilsson and G. Como, "A micro-simulation study of the generalized proportional allocation traffic signal control," *IEEE Transactions on Intelligent Transportation Systems*, 2020. [Online]. Available: https://doi.org/10.1109/TITS.2019.2957718

[22] G. Como, "On resilient control of dynamical flow networks," *Annual Reviews in Control*, vol. 43, pp. 80 – 90, 2017.