

# A Pricing Mechanism for Balancing the Charging of Ride-Hailing Electric Vehicle Fleets

Marko Maljkovic, Gustav Nilsson, and Nikolas Geroliminis

**Abstract**—Both ride-hailing services and electric vehicles are becoming increasingly popular and it is likely that charging management of the ride-hailing vehicles will be a significant part of the ride-hailing company’s operation in the near future. Motivated by this, we propose a game theoretic model for charging management, where we assume that it is the fleet-operator that wants to minimize its operational cost, which among others include the price of charging. To avoid overcrowded charging stations, a central authority will design pricing policies to incentivize the vehicles to spread out among the charging stations, in a setting where several ride-hailing companies compete about the resources. We show that it is possible to construct pricing policies that make the Nash-equilibrium between the companies follow the central authority’s target value when the desired load is feasible. Moreover, we provide a decentralized algorithm for computation of the equilibrium and conclude the paper with a numerical example illustrating the results.

## I. INTRODUCTION

Ride-hailing services have become more and more popular over the past years and are nowadays an essential part of transportation services in many cities. Also, electric vehicles (EVs) are becoming more common and are soon likely to become a significant part of the fleets of vehicles that ride-hailing companies manage. Since ride-hailing companies already offer access to cleaning and service stations to their drivers, it is not unlikely that in the future, they will offer discounted charging as well. By doing so, the companies can gain control of both the coverage by sending vehicles to charge in areas where there is demand and the availability, e.g., by incentivizing the drivers to charge up their vehicles before demand peaks. Moreover, given the asymmetric distribution of origins and destinations, pricing incentives can contribute also in rebalancing vehicles in regions of higher demand. In the case of autonomous fleets, the ride-hailing company would have total control of the vehicles and could also fully control the charging.

Inspired by this vision, this paper presents a pricing mechanism to load balance the ride-hailing vehicles among different charging stations. We study a scenario where a central body, e.g., the government of the city or the power providing company, defines the set points describing how

M. Maljkovic, G. Nilsson, and N. Geroliminis are with the School of Architecture, Civil and Environmental Engineering, École Polytechnique Fédérale de Lausanne (EPFL), 1015 Lausanne, Switzerland. {marko.maljkovic, gustav.nilsson, nikolas.geroliminis}@epfl.ch.

This work was supported by the Swiss National Science Foundation under NCCR Automation, grant agreement 51NF40.180545.

An extended version containing all the proofs is available at <http://arxiv.org/abs/2203.09327>

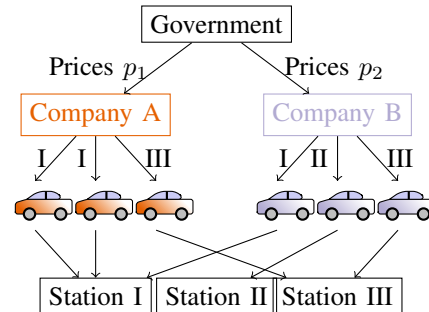


Fig. 1. Schematic sketch of the problem setting. The central body, e.g., the government or the power company, wants to balance the vehicle load on different charging stations through pricing policies. With the provided pricing policies, each ride-hailing company wants to minimize its own operational cost by steering its vehicles to different charging stations.

the vehicles should spread out among the charging facilities in an attempt to either help fight the congestion in the city or to balance the demand on the power grid. The central body is incentivizing the ride-hailing companies to follow the desired set points through pricing while each company is trying to optimize its operational cost by directing its vehicles to different charging stations. A schematic representation of the problem is shown in Figure 1. Due to every company’s interest in minimizing its queuing time at the stations, there is an inherent competition among them establishing fertile ground for game theoretic analysis.

Research has shown that the frameworks of congestion, mean-field, Stackelberg, and inverse Stackelberg games are powerful tools for solving problems within the realm of transportation and mobility systems. In [1], [2], through congestion game based routing, tolling mechanisms have been designed for congestion control of urban networks, whereas in [3], charging station allocation for a population of EVs has been performed. The structure of the mean-field games, where the utility of each player depends on the aggregation of other players’ decisions, offers a suitable setting for charging control of a population of EVs as presented in [4]–[7]. Our work is similar to [4]–[7] in a sense that the underlying structure of our problem can also be described by an aggregative game. However, we also go along the line of research that focuses on Stackelberg and inverse Stackelberg games to design pricing and tolling mechanisms primarily for revenue maximization. In [8], [9], the charging stations act as revenue maximizing leaders in a Stackelberg game, whereas individual EVs act as charging cost minimizing followers. The setup in [8], [9] assumes

fixed optimal prices of charging. In this paper, we propose using a pricing mechanism based on the decision of the ride-hailing companies which allows us to directly influence the placement of the Nash equilibrium. This makes our setup more similar to the ones presented in [10], [11] where inverse Stackelberg game has been used to solve hierarchical control and bi-level optimal toll design problems. The inverse Stackelberg pricing schemes are different from the Stackelberg ones in a sense that the prices are not a priori set to a certain value by the leading player, i.e., the central body, but are rather announced as a function of the followers' decisions. This means that the companies do not know what the charging prices will be before they make a decision on how to direct their vehicles but rather how their joint decision will influence the prices of charging.

To the best of our knowledge, no work so far has provided a comprehensive framework for analysing the problem of balancing the charging of EV fleets operated by ride-hailing companies so as to achieve the objective of a higher level authority. Moreover, we do so in a decentralized manner, with little private information exchange between the ride-hailing companies and the government and under the reachability constraints imposed by the state of the individual car fleets.

The paper is outlined as follows: the rest of this section is devoted to introducing some basic notation. In Section II we introduce the model and state the main formulation. In the following section, Section III, we present the pricing mechanism and show that this pricing mechanism achieves a unique Nash-equilibrium between the companies. We also provide an algorithm to compute the Nash-equilibrium. In Section IV, we illustrate the proposed solution through a numerical example and conclude the paper with some ideas for future research in Section V.

### A. Notation

Let  $\mathbb{R}$  denote the set of real numbers, and  $\mathbb{R}_+$  the set of non-negative reals. Let  $\mathbf{0}_m$  and  $\mathbf{1}_m$  denote the all zero and all one vectors of length  $m$  respectively, and  $\mathbb{I}_m$  the identity matrix of size  $m \times m$ . For a finite set  $\mathcal{A}$ , we let  $\mathbb{R}_{(+)}^{\mathcal{A}}$  denote the set of (non-negative) vectors indexed by the elements of  $\mathcal{A}$ ,  $|\mathcal{A}|$  the cardinality of  $\mathcal{A}$  and we let  $\mathcal{P}_{\mathcal{A}}$  be the probability space over the set, i.e.,  $\mathcal{P}_{\mathcal{A}} := \{x \in \mathbb{R}_+^{\mathcal{A}} \mid \sum_{i \in \mathcal{A}} x_i = 1\}$ . For a diagonal matrix  $A \in \mathbb{R}^{n \times n}$ , we let  $A^*$  denote its pseudo-inverse, i.e.,

$$A_{ii}^* := \begin{cases} 1/A_{ii} & \text{if } A_{ii} \neq 0, \\ 0 & \text{otherwise,} \end{cases} \quad 1 \leq i \leq n.$$

## II. MODEL

We consider a setting where different ride-hailing companies have access to common charging stations for their electric vehicles. We let  $\mathcal{C}$  denote the set of companies, and  $N_i > 0$  the number of vehicles belonging to each company  $i \in \mathcal{C}$ . The vector of the number of vehicles for all companies is denoted by  $N \in \mathbb{R}_+^{\mathcal{C}}$ . We let  $\mathcal{M}$  represent the set of charging stations, and  $M_j > 0$  the number of spots available at each charging station  $j \in \mathcal{M}$ , i.e., the charging station's

capacity. The vector of all charging stations capacities is denoted by  $M \in \mathbb{R}_+^{\mathcal{M}}$  and the cardinality of  $\mathcal{M}$  as  $m = |\mathcal{M}|$ .

For each company  $i \in \mathcal{C}$ , we let  $\mathcal{V}_i$  be the set of its vehicles with  $|\mathcal{V}_i| = N_i$  and  $x^i \in \mathcal{P}_{\mathcal{M}}$  denote the fraction of vehicles that the company wants to send to each charging station, i.e.,  $x_j^i$  is the fraction of vehicles from company  $i \in \mathcal{C}$  that will be sent to charging station  $j \in \mathcal{M}$ . Furthermore, let  $n_j^i \in \mathbb{Z}_+$  denote the integer number of vehicles, associated with the continuous allocation  $x_j^i$ , that the operator of the fleet would send to station  $j$ . Since not all charging stations are reachable for all vehicles and hence not all choices of  $x^i$  are feasible, we define for each company the feasibility sets  $\mathcal{F}_j^i := \{v \in \mathcal{V}_i \mid v \text{ can reach station } j\}$ .

We say that a continuous allocation vector  $x^i$  is feasible, if it allows the operator of the company to choose any discrete allocation  $n^i \in \mathbb{Z}_+^{\mathcal{M}}$  where individual  $n_j^i$  can be either  $\lfloor N_i x_j^i \rfloor$  or  $\lceil N_i x_j^i \rceil$  under the constraints that  $\sum_{j \in \mathcal{M}} n_j^i = N_i$  and that there exists a feasible matching between the vehicles and the charging stations for the chosen  $n^i$ . For each company  $i \in \mathcal{C}$ , we let  $\mathcal{K}_i \subseteq \mathcal{P}_{\mathcal{M}}$  denote the set of all feasible  $x^i$ . Furthermore, we define  $\mathcal{K} := \prod_{i \in \mathcal{C}} \mathcal{K}_i$  and  $\mathcal{K}_{-i} := \prod_{j \in \mathcal{C} \setminus i} \mathcal{K}_j$ .

We let  $x := [x^i]_{i \in \mathcal{C}} \in \mathcal{K}$  denote all companies' decision vectors,  $x^{-i} := [x^j]_{j \in \mathcal{C} \setminus i} \in \mathcal{K}_{-i}$  denote the decision vectors of all companies except the company  $i$ ,  $\sigma(x) := \sum_{i \in \mathcal{C}} N_i x^i \in \mathbb{R}_+^{\mathcal{M}}$  denote the vector consisting of the total number of vehicles that have chosen each station and  $\sigma(x^{-i}) := \sum_{j \in \mathcal{C} \setminus i} N_j x^j \in \mathbb{R}_+^{\mathcal{M}}$  denote the vector consisting of the number of vehicles from the other companies that have chosen each station.

To easily distinguish between the agents, we refer to the central authority as the "government". It is interested in balancing the vehicles so as to minimize the personal objective of the form

$$J_G(\sigma(x)) = \frac{1}{2} \sigma(x)^T A_G \sigma(x) + b_G^T \sigma(x), \quad (1)$$

for some diagonal matrix  $A_G \succ 0$  and  $b_G \in \mathbb{R}^{\mathcal{M}}$ . In this paper, we are particularly interested in balancing the vehicles so that the number of vehicles charging at each station equals  $\hat{N}$ , i.e., to minimize

$$J_G(\sigma(x)) = \frac{1}{2} \|\sigma(x) - \hat{N}\|_{2, A_G}^2, \quad (2)$$

where  $A_G$  gives the government the possibility to penalize deviations from the desired number of vehicles differently at different stations. It should be noted that (2) is a special case of (1) that can be obtained by letting in  $b_G = -A_G \hat{N} \in \mathbb{R}^{\mathcal{M}}$ .

To steer the companies to the minimum of (2), the government will assign an individual pricing policy to each company for each charging station. The policy will be a function of the choice of the company itself but also of the other companies' choices, since the government's interest is to control the total number of vehicles. For company  $i \in \mathcal{C}$ , the pricing policy is  $p_i(x^i, x^{-i}) : \mathcal{K}_i \times \mathcal{K}_{-i} \rightarrow \mathbb{R}^{\mathcal{M}}$ .

After the pricing policies are announced, the government and the companies admit an inverse Stackelberg game in

which every company is trying to minimize its own operational cost, under the constraint that all the company's vehicles must be able to reach a charging station. We model the operational cost for each company as a sum of three terms. The first term, denoted as the queuing cost, depends on the choice of the company itself but also on the cumulative choice of all other companies and has the general form

$$J_1^i(x^i, \sigma(x)) = \frac{1}{2}(x^i)^T A_i x^i + (x^i)^T B_i \sigma(x^{-i}) + c_i^T x^i, \quad (3)$$

for some diagonal matrices  $A_i \in \mathbb{R}^{\mathcal{M} \times \mathcal{M}}$ ,  $B_i \in \mathbb{R}^{\mathcal{M} \times \mathcal{M}}$  and  $c_i \in \mathbb{R}^{\mathcal{M}}$ . In this paper, we model the expected queuing cost as  $J_1^i(x^i, \sigma(x)) = N_i (x^i)^T Q (\sigma(x) - M)$ , which is a special case of (3) if we set  $A_i := 2N_i^2 Q$ ,  $B_i := N_i Q$  and  $c_i := -N_i Q M$ . Here,  $Q \in \mathbb{R}^{\mathcal{M} \times \mathcal{M}}$  is a positive definite diagonal scaling matrix whose diagonal entries describe how expensive it is to queue in the regions around charging stations. Generally, more congested areas should experience higher queuing costs and hence higher scaling factors. We model the second term which describes the charging cost as a function of the choice of the company and the pricing policy assigned to it, i.e.,  $J_2^i(x^i, p_i(x^i, x^{-i})) = (x^i)^T D_i p_i(x^i, x^{-i})$ , for some diagonal  $D_i \succeq 0$ . The diagonal entry  $(D_i)_{kk}$  can be interpreted as the part of the total charging demand to be served at the charging station  $k$ . The third term we denote as the negative expected revenue and model it as a function of only the company's choices, i.e.,  $J_3^i(x^i) = f_i^T x^i$ . Here, we interpret the negative expected revenue as the difference between the cost of fleet being idle while traveling to the charging stations and the expected profit in the regions around charging stations after the charging has been completed. The information about the negative expected revenue per vehicle is encoded in  $f_i$ . Hence, the company cost can be in general expressed as

$$J^i(x^i, x^{-i}) = J_1^i(x^i, \sigma(x)) + J_2^i(x^i, p_i(x^i, x^{-i})) + J_3^i(x^i) \quad (4)$$

and each company  $i \in \mathcal{C}$  would like to allocate its vehicles according to

$$x^{i*} \in \arg \min_{x^i \in \mathcal{K}_i} J^i(x^i, x^{-i}). \quad (5)$$

We say that the government and the companies admit a system optimum if there exists  $x^*$  that minimizes (1) and satisfies (5) for all  $i \in \mathcal{C}$ . We will show in the following section that if we can reduce the decision space of the companies to convex subsets  $\bar{\mathcal{K}}_i \subseteq \mathcal{K}_i$ , under the proposed pricing strategies there will be a unique system optimum.

To summarize, we consider the problem of designing prices, such that each company will steer its fleet of vehicles towards predefined target values of vehicle accumulations around different charging stations. A schematic sketch of the problem is shown in Figure 1 and the problem is formally stated below.

*Problem 1:* Design pricing policies  $p_i(x^i, x^{-i})$  and the constraint sets  $\bar{\mathcal{K}}_i \subseteq \mathcal{K}_i$  such that there is a unique Nash

equilibrium of the game  $G$  defined as

$$G := \left\{ \min_{x^i \in \bar{\mathcal{K}}_i} J^i(x^i, x^{-i}), \forall i \in \mathcal{C} \right\}, \quad (6)$$

with  $J^i$  defined as in (4). Moreover, the Nash equilibrium should be such that it also minimizes the government cost  $J_G(x)$  in (1) and the design of the constraint sets  $\bar{\mathcal{K}}_i$  such that existence of a feasible discrete allocation scheme for each company is guaranteed.

### III. PRICING MECHANISM

We begin this section by showing how the sets  $\bar{\mathcal{K}}_i$  can be constructed. With the existence of those sets, we then proceed to introduce a pricing policy that achieves a unique Nash equilibrium for allocating the vehicles of all companies. Moreover, we show that this Nash equilibrium also minimizes the government's cost function which makes it a unique system optimum. In the last part of the section, we propose an algorithm for computing the Nash equilibrium.

In the following proposition, we show how to analytically construct convex sets  $\bar{\mathcal{K}}_i \subseteq \mathcal{K}_i$  based on feasibility sets  $\mathcal{F}_j^i$ , that guarantee feasibility of  $x^i$  defined as in Section II.

*Proposition 1:* For each company  $i \in \mathcal{C}$ , define the set  $\bar{\mathcal{K}}_i \subseteq \mathcal{K}_i \subseteq \mathcal{P}_{\mathcal{M}}$  such that  $x^i \in \bar{\mathcal{K}}_i$  if for all proper subsets  $S$  of  $\mathcal{M}$ , it holds that

$$N_i \sum_{j \in S} x_j^i \leq \max \left\{ 0, \left| \bigcup_{j \in S} \mathcal{F}_j^i \right| - |S| \right\}. \quad (7)$$

If the state of the car fleet does not correspond to a degenerate case for which  $\bar{\mathcal{K}}_i = \emptyset$ , then every  $x^i \in \bar{\mathcal{K}}_i$  is feasible and  $\bar{\mathcal{K}}_i$  is compact and convex.

The proof of Proposition 1 is given in the extended version of our paper.

*Remark 1:* For every subset  $S$  of  $\mathcal{M}$ , a constraint on the discrete allocation vector  $n^i$  given by

$$\sum_{j \in S} n_j^i \leq \left| \bigcup_{j \in S} \mathcal{F}_j^i \right|, \quad (8)$$

must be fulfilled so that every vehicle is matched with exactly one charging station. Intuitively, inequality (8) states that for any subset of the charging stations, the operator of the company must not allocate more vehicles than what is feasible. In fact, the constraint on the continuous allocation vector  $x^i$  given by (7) is a tightened version of the constraint (8) that guarantees the condition (8) will be fulfilled regardless of how the operator chooses  $n^i$  based on  $x^i$ . Degenerate states of the car fleet that result in  $\bar{\mathcal{K}}_i = \emptyset$  correspond to cases where most of the vehicles have very limited options when choosing the station to charge and as such are not the subject of our interest.

Let  $\bar{\mathcal{K}} := \prod_{i \in \mathcal{C}} \bar{\mathcal{K}}_i$  and  $\bar{\mathcal{K}}_{-i} := \prod_{j \in \mathcal{C} \setminus i} \bar{\mathcal{K}}_j$ . We will now introduce our pricing mechanism.

*Definition 1 (System Optimal Pricing Policies):* For each company  $i \in \mathcal{C}$ , let

$$p_i(x^i, x^{-i}) = D_i^* \left[ \frac{1}{2} \bar{A}_i x^i + \bar{B}_i \sigma(x^{-i}) + \Delta_i \right]. \quad (9)$$

where  $\bar{A}_i = N_i^2 A_G - A_i$ ,  $\bar{B}_i = N_i A_G - B_i$  and  $\Delta_i = N_i b_G - c_i - f_i$ .

*Remark 2:* For a company  $i \in \mathcal{C}$ , unreachable stations will correspond to zero diagonal entries in the matrix  $D_i$ , which makes the matrix not invertible. However, since company  $i$  will not use those charging stations, letting the prices for those stations be zero through the pseudo-inverse will not affect the solution of the problem.

We will later in this section show that these pricing policies minimize the government's objective, which explains why we refer to the pricing policies as system optimal.

Next, we will show that the proposed pricing policies will give raise to a unique Nash equilibrium in the game between the companies.

*Theorem 1:* For all companies  $i \in \mathcal{C}$ , let the sets  $\bar{\mathcal{K}}_i$  be designed as in Proposition 1. Then, with the system optimal pricing policies in Definition 1, the game  $G$  in (6) has a unique Nash equilibrium.

**Proof** To prove existence and uniqueness of the Nash equilibrium, we rely on techniques from [12]. Inserting policy (9) into (4), and utilizing that for  $x^i \in \bar{\mathcal{K}}_i$  it holds that  $D_i D_i^* x^i = x^i$ , transforms the cost of each company  $i \in \mathcal{C}$  into  $J^i(x^i, x^{-i}) = \frac{1}{2} (x^i)^T N_i^2 A_G x^i + (x^i)^T N_i (A_G \sigma(x^{-i}) + b_G)$ . Since the action spaces  $\bar{\mathcal{K}}_i$  are compact (as a subset of the probability space over  $\mathcal{M}$ ), convex and satisfy Slater's constraint qualification by construction,  $J^i(x)$  are continuous in  $x \in \bar{\mathcal{K}}$ ,  $J^i(x^i, x^{-i})$  are convex in  $x^i \in \bar{\mathcal{K}}_i$  for a fixed  $x^{-i} \in \bar{\mathcal{K}}_{-i}$  and players perform minimization of the objective, [12, T.1] guarantees existence of a Nash equilibrium. According to [12, T.2], a sufficient condition for the Nash equilibrium to be unique is that the symmetric matrix  $\Gamma := G(x, r) + G^T(x, r)$  be negative definite for  $x \in \bar{\mathcal{K}}$  and some  $r = [r_i]_{i \in \mathcal{C}} \in \mathbb{R}_{>0}^{\mathcal{C}}$  with  $G(x, r)$  being the Jacobian with respect to  $x$  of function  $g(x, r)$  defined as  $g(x, r) := [-r_i \nabla_{x^i} J^i(x)]_{i \in \mathcal{C}}$ . For  $r = \mathbf{1}_{|\mathcal{C}|}$  and any  $x \in \bar{\mathcal{K}}$  we have  $x^T \Gamma x = -2 (\sum_{i \in \mathcal{C}} N_i x^i)^T A_G (\sum_{i \in \mathcal{C}} N_i x^i)$ .

Since  $\bar{\mathcal{K}}_i \subseteq \mathcal{P}_{\mathcal{M}}$  for all  $i \in \mathcal{C}$ , we have  $\sum_{i \in \mathcal{C}} N_i x^i \neq \mathbf{0}_{|\mathcal{M}|}$ . Since  $A_G \succ 0$ , we have  $x^T \Gamma x < 0$  for all  $x \in \bar{\mathcal{K}}$  which proves that  $\Gamma$  is negative definite on  $\bar{\mathcal{K}}$  and that the Nash equilibrium is unique. ■

Now that we know that under the pricing policies given by (9) the Nash equilibrium is unique, we proceed to show that it also minimizes the government objective (1).

*Theorem 2:* For all companies  $i \in \mathcal{C}$ , let the sets  $\bar{\mathcal{K}}_i$  be designed as in Proposition 1. Then, with the system optimal pricing policies in Definition 1, the Nash equilibrium  $x^*$  of game  $G$  satisfies

$$x^* \in \arg \min_{x \in \bar{\mathcal{K}}} J_G(\sigma(x)).$$

**Proof** Let  $A^T := [N_i \mathbb{1}_{|\mathcal{M}|}]_{i \in \mathcal{C}} \in \mathbb{R}^{|\mathcal{M}| \times |\mathcal{C}| \times |\mathcal{M}|}$ , then the government optimization problem is equivalent to

$$\min_{x \in \bar{\mathcal{K}}} J_G(x) := \frac{1}{2} x^T A^T A_G A x + b_G^T A x. \quad (10)$$

The function  $J_G(x)$  is convex since  $\nabla_x^2 J_G(x) = A^T A_G A$  and for all  $x^i \in \bar{\mathcal{K}}_i \subseteq \mathcal{P}_{\mathcal{M}}$  it holds that  $\sum_{i \in \mathcal{C}} N_i x^i \neq \mathbf{0}_{|\mathcal{M}|}$  so  $x^T A^T A_G A x = (\sum_{i \in \mathcal{C}} N_i x^i)^T A_G (\sum_{i \in \mathcal{C}} N_i x^i) > 0$  which guarantees that  $\nabla_x^2 J_G(x) \succeq 0$ . According to [13, 4.21],  $x^*$  is the minimizer of (10) on  $\bar{\mathcal{K}}$  if and only if  $\langle \nabla_x J_G(x) |_{x=x^*}, y - x^* \rangle \geq 0, \forall y \in \bar{\mathcal{K}}$ . Under the pricing policies defined in (9),  $J_G(x)$  is the exact potential [14] for game  $G$  satisfying for all  $i \in \mathcal{C}$  and any fixed  $x^{-i} \in \bar{\mathcal{K}}_{-i}$

$$\nabla_{x^i} J^i(x^i, x^{-i}) = \nabla_{x^i} J_G(x^i, x^{-i}), \quad \forall x^i \in \bar{\mathcal{K}}_i. \quad (11)$$

If  $\hat{x}$  is the Nash equilibrium of  $G$ , then for all  $i \in \mathcal{C}$  we have  $\hat{x}^i \in \arg \min_{x^i \in \bar{\mathcal{K}}_i} J^i(x^i, \hat{x}^{-i})$ . According to [13], we can now write  $\langle \nabla_{x^i} J^i(x) |_{x=\hat{x}}, y^i - \hat{x}^i \rangle \geq 0$ , for all  $y^i \in \bar{\mathcal{K}}_i$  for all  $i \in \mathcal{C}$ . Because of (11), for all  $i \in \mathcal{C}$  and for all  $y^i \in \bar{\mathcal{K}}_i$  it holds that  $\sum_{i \in \mathcal{C}} \langle \nabla_{x^i} J_G(x) |_{x=\hat{x}}, y^i - \hat{x}^i \rangle \geq 0$ . Finally, we have that  $\hat{x}$  indeed is the minimizer of (10) since  $\sum_{i \in \mathcal{C}} \langle \nabla_{x^i} J_G(x) |_{x=\hat{x}}, y^i - \hat{x}^i \rangle = \langle \nabla_x J_G(x) |_{x=\hat{x}}, y - \hat{x} \rangle$  is true for any  $y \in \bar{\mathcal{K}}$ . ■

Sets  $\bar{\mathcal{K}}_i$  defined as (7) reflect the current state of the car fleets. In realistic scenarios, these sets are private, i.e., not known to the government, as they encompass information about the current true location of the vehicles and their current and desired battery status, preventing centralized computation of the Nash equilibrium. Hence, a decentralized algorithm with minimal exchange of information between the agents is required. Such algorithms based on theory of aggregative games were proposed in [4], [5] and [15]. Based on [15], since our game-map, defined as  $F(x) = [\nabla_{x^i} J^i(x^i, x^{-i})]_{i \in \mathcal{C}}$ , is equal to

$$F(x) = \underbrace{A^T A_G A x}_{F_1} + \underbrace{A^T b_G}_{F_2} \quad (12)$$

and is a non-strictly monotonic ( $F_1 \succeq 0$ ) linear operator, we utilize a distributed iterative scheme based on the Krasnosel'skij iteration [16] to find the Nash equilibrium of  $G$ .

*Proposition 2:* Under the system optimal pricing policies and for sets  $\bar{\mathcal{K}}_i$  as in Proposition 1, for every  $\gamma$  such that

$$0 < \gamma < \frac{2}{\lambda_{\max}(F_1)}, \quad (13)$$

a distributed iterative scheme given by

$$x^i(k+1) = \frac{1}{2} (x^i(k) + \Pi_{\bar{\mathcal{K}}_i} [x^i(k) - \gamma \nabla_{x^i} J^i(x^i, x^{-i})]),$$

where  $\Pi_{\bar{\mathcal{K}}_i}$  denotes the projection operator onto  $\bar{\mathcal{K}}_i$ , converges to the Nash equilibrium of the game  $G$ .

**Proof** A point  $\hat{x} \in \bar{\mathcal{K}}$  is a Nash equilibrium of game  $G$  with game map  $F(x)$  defined by (12) if and only if  $F(\hat{x})^T (y - \hat{x}) \geq 0$  for all  $y \in \bar{\mathcal{K}}$  [17]. One can prove that  $F(\hat{x})^T (y - \hat{x}) \geq 0$  holds for all  $y \in \bar{\mathcal{K}}$  if and only if  $\hat{x} = \Pi_{\bar{\mathcal{K}}} [\hat{x} - \gamma F(\hat{x})]$ . Indeed, based on [13]

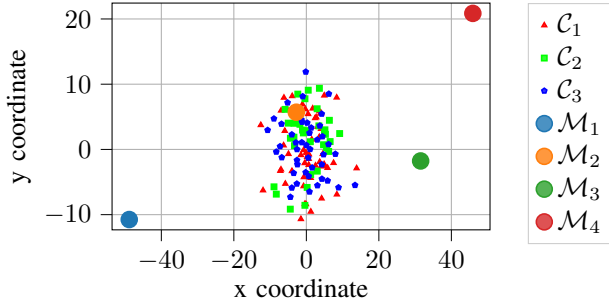


Fig. 2. Shows the location of charging station 1 ( $\mathcal{M}_1$ ), station 2 ( $\mathcal{M}_2$ ), station 3 ( $\mathcal{M}_3$ ) and station 4 ( $\mathcal{M}_4$ ). Moreover, black triangles, squares and pentagons show the locations of vehicles that belong to company 1 ( $\mathcal{C}_1$ ), company 2 ( $\mathcal{C}_2$ ) and company 3 ( $\mathcal{C}_3$ ) respectively.

and the fact that  $z = \Pi_{\bar{\mathcal{K}}}[\hat{x} - \gamma F(\hat{x})]$  is equivalent to  $z$  being the minimizer of  $\|z - (\hat{x} - \gamma F(\hat{x}))\|_2^2$  over  $\bar{\mathcal{K}}$ , we have that  $z = \Pi_{\bar{\mathcal{K}}}[\hat{x} - \gamma F(\hat{x})]$  is equivalent to  $2(z - (\hat{x} - \gamma F(\hat{x})))^T (y - z) \geq 0$ , for all  $y \in \bar{\mathcal{K}}$ , which by setting  $z = \hat{x}$  completes the proof of equivalence. Now we have that  $\hat{x}$  is a Nash equilibrium if and only if it is a fixed point of  $H(x) = \Pi_{\bar{\mathcal{K}}}[x - \gamma F(x)]$ . Because  $\bar{\mathcal{K}}$  is compact and convex, for any  $\gamma$  such that  $H(x)$  is non-expansive and  $x(0) \in \bar{\mathcal{K}}$ , the iterative procedure  $x(k+1) = 0.5(x(k) + H(x(k)))$  converges to the fixed point of  $H(x)$  (the unique Nash equilibrium of  $G$ ) according to [16]. Since the projection operator is non-expansive, for  $\gamma$  such that  $\bar{H}(x) = \mathbb{I}x - \gamma F(x) = (\mathbb{I} - \gamma F_1)x - \gamma F_2$  is non-expansive, the map  $H(x)$  will be non-expansive too.  $\bar{H}(x)$  is an affine map so it is non-expansive if  $\|\mathbb{I} - \gamma F_1\|_2 \leq 1$ . Since  $F_1$  is symmetric this is equivalent to  $\max_i |\lambda_i(\mathbb{I} - \gamma F_1)| \leq 1$ . For  $\gamma$  given in (13) and because  $F_1 \succeq 0$  this is guaranteed since  $-1 \leq 1 - \gamma \lambda_i(F_1) \leq 1$  is true for all  $i$ . ■

#### IV. NUMERICAL EXAMPLE

We illustrate in this section how the proposed method can be utilized to balance the EVs so that the number of them charging at different stations is as close as possible to vector  $\tilde{N}$ . We consider a scenario where 3 ride-hailing companies  $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$ , whose fleet sizes are given by  $N = [60, 35, 45]^T$ , operate in a square region with 4 charging stations  $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4\}$ . The stations are described by the vector of their capacities  $M = [20, 10, 15, 10]^T$  and we set desired vehicle numbers around them to be  $\tilde{N} = [35, 15, 50, 40]^T$ .

Each vehicle  $v_j \in V_i$  is described by a tuple  $(x_j, y_j, s_j^{\text{start}}, s_j^{\text{des}}, d_j^{\text{max}})$  where  $(x_j, y_j) \in \mathbb{R}^2$  describes the position of the vehicle,  $d_j^{\text{max}}$  is the max range of the vehicle and  $s_j^{\text{start}}, s_j^{\text{des}}$  represent the current and desired battery levels. The vehicles and charging stations are placed randomly with:  $s_j^{\text{start}} \sim \mathcal{U}[20, 40]$ ,  $s_j^{\text{des}} \sim \mathcal{U}[80, 100]$  and  $d_j^{\text{max}} \sim \mathcal{U}[150, 200]$ . The scenario is depicted in Figure 2. A station is considered to be feasible to a vehicle if the vehicle can reach it with the current battery status. For simplicity, if we assume a linear battery discharge model, a charging station  $k$  is feasible for vehicle  $j$  if  $s_j^{\text{start}} - \frac{100}{d_j^{\text{max}}} d_{j,k} > 0$  where

TABLE I  
COMPANY DECISIONS AND CHARGING PRICES

$\mathcal{C}$	Station 1		Station 2		Station 3		Station 4	
	$x_1^i$	$p_i$	$x_2^i$	$p_i$	$x_3^i$	$p_i$	$x_4^i$	$p_i$
$\mathcal{C}_1$	0.20	1.65	0.15	3.78	0.38	1.16	0.27	0.98
$\mathcal{C}_2$	0.19	1.75	0.16	4.12	0.41	1.48	0.24	1.17
$\mathcal{C}_3$	0.21	1.77	0.10	4.16	0.43	1.29	0.26	1.04

$d_{j,k}$  denotes the distance between the vehicle  $j$  and the charging station  $k$  and  $s_j^{\text{start}}$  is expressed in percentage. The average charging cost is modelled as  $J_2^i(x^i, p_i(x^i, x^{-i})) = N_i(x^i)^T R_i p_i(x^i, x^{-i})$ . Diagonal matrix  $R_i \in \mathbb{R}^{4 \times 4}$  captures the average charging demand per vehicle when choosing each of the charging stations. For infeasible charging stations the average demand is set to 0. Pricing policy  $p_i$  denotes the price of one unit of charge at each station. If the charging station  $k$  is feasible to vehicle  $v_l \in V_i$ , vehicle's charging demand if  $k$  is chosen for charging is defined as  $\delta_{l,k} = \beta_l \left( s_l^{\text{des}} - \left( s_l^{\text{start}} - \frac{100}{d_l^{\text{max}}} d_{l,k} \right) \right)$ . Here  $\beta_l \in \mathbb{R}$  is a scaling coefficient that says how many units of charge corresponds to 1% of the vehicle's battery. The diagonal element of  $R_i$  that corresponds to station  $k$  is then given by  $(R_i)_{kk} = \left[ \frac{1}{|\mathcal{F}_k^i|} \sum_{l: v_l \in \mathcal{F}_k^i} \delta_{l,k} \right]$ . We model the negative expected revenue as  $J_3^i(x^i) = (e_i^{\text{arr}})^T N_i x^i - (e_i^{\text{pro}})^T N_i x^i$ . Here,  $e_i^{\text{arr}} \in \mathbb{R}^4$  is the average cost of a vehicle being unoccupied while traveling to a charging station. If station  $k$  is infeasible, then we set  $(e_i^{\text{arr}})_k = 0$ , otherwise it is equal to  $(e_i^{\text{arr}})_k = u_i \cdot P_k \cdot \left[ \frac{1}{|\mathcal{F}_k^i|} \sum_{l: v_l \in \mathcal{F}_k^i} d_{l,k} \right]$ , where  $u_i \in \mathbb{R}$  is the monetary value of a vehicle being occupied while driving for 1km, given in [\$/km] and  $P_k$  is the probability of a vehicle being occupied in the region around charging station  $k$ . The vector  $e_i^{\text{pro}} \in \mathbb{R}^4$  denotes expected profit in regions around different charging stations. In general, this vector is obtained from historical data and here we choose it randomly such that each element of  $e_i^{\text{pro}}$  satisfies  $e_j^{\text{pro}} \sim \mathcal{U}[100, 350]$ . The sample drawn in this simulation is  $e_i^{\text{pro}} = [202.51, 301.02, 252.34, 195.61]^T$ . We fix other parameters to  $\beta_l = 1.0$ ,  $Q = \text{diag}(1, 5, 3, 2)$  and  $A_G = 2Q$ , vector of probabilities of being occupied  $P = [0.15, 0.4, 0.2, 0.1]$  for all  $k \in \mathcal{M}$ ,  $u_i = 1.0$  for all  $i \in \mathcal{C}$ , and set the number of iterations for the algorithm to  $k = 3000$ . For this case study, the optimal pricing policy in accordance with (9) is obtained by setting  $D_i := N_i R_i$  and  $f_i := N_i (e_i^{\text{arr}} - e_i^{\text{pro}})$ .

In the Nash equilibrium, car fleet portions to be directed to each of the charging stations and the resulting charging prices are presented in Table I whereas the evolution of the government loss  $J_G$  and the total number of vehicles over the iterations is presented in Figure 3. From the plot it is clear that the iterative procedure converged to a Nash equilibrium that is the government optimum but does not perfectly match the predefined vehicle accumulation vector  $\tilde{N}$  due to vehicle arrangement and their battery status. As expected, the prices of charging at station 2 are significantly higher than for

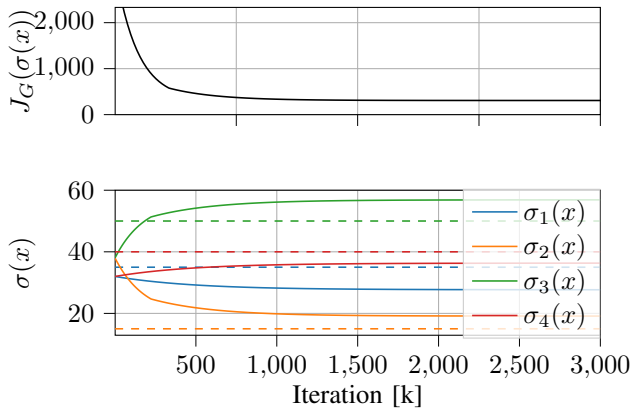


Fig. 3. The evolution of the government loss function and the total number of vehicles to be charged at each station during the iterative procedure for finding the Nash equilibrium. Dotted lines represent the target values. The execution time of all simulations on an average PC was smaller than 3 sec.

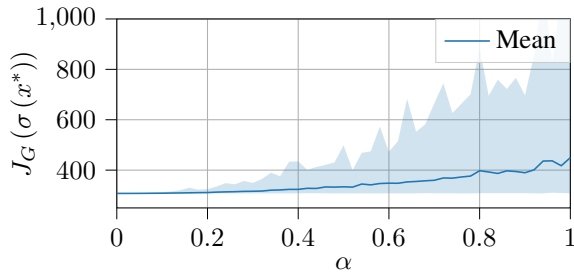


Fig. 4. When the government does not have correct information about the vehicles' locations and what their charging demands are, the pricing policies divert from the optimal ones. For each  $\alpha$ , the mean value of  $J_G$  is plotted over 100 simulations, together with its maximum and minimum value.

any other charging station for all the companies as it has the smallest desired vehicle accumulation and is the most desirable in terms of expected profit and the distance to be travelled to reach it. Station 4 is the least attractive hence, it has the smallest charging prices in the Nash equilibrium.

Apart from  $R_i$  and  $e^{\text{arr}}$ , all other parameters are inherently known to the government as they characterize the region in which the companies operate. Hence, the government optimum is attainable if the companies are willing to share  $R_i$  and  $e^{\text{arr}}$  that encompass the information about the average state of the company's fleet. We test robustness of the proposed pricing policies and show how the system behaves in the same scenario when the government has only an estimate  $\bar{R}_i$  of the average charging demand  $R_i$ . For a feasible station  $k$ , we let  $(\bar{R}_i)_{kk} = (R_i)_{kk} + w_k$  where  $w_k$  is a noise sample such that  $w_k \sim \mathcal{N}(0, (\alpha R_{\min}/5)^2)$  with  $R_{\min}$  being the minimal, non-zero, diagonal element of  $R_i$ . For every  $\alpha$  we sample  $w_k$  one hundred times and report the mean value of the government's loss in the Nash equilibrium. Figure 4 shows that for moderate discrepancies ( $\alpha < 0.6$ ) between the true and the estimated value of  $R_i$ , the attained Nash equilibrium is close to the government's optimum. It also confirms that the worse the approximation is, the higher the deviation of the Nash equilibrium from  $\hat{N}$  will be.

## V. CONCLUSIONS

In this paper we have developed a model for charge pricing of fleets of electric ride-hailing vehicles, where a central authority wants to control the demand on the charging stations through pricing. We constructed a set of pricing policies, and showed that those policies both give rise to a unique Nash equilibrium when each fleet operator wants to minimize its own operational cost and that this Nash equilibrium also minimizes the deviation from the central authority's desire.

In the future, we plan to deeper address the robustness of the proposed solution, something that is needed when the government does not have full knowledge of vehicles' position and charging demands.

## REFERENCES

- [1] J. Zhang, J. Lu, J. Cao, W. Huang, J. Guo, and Y. Wei, "Traffic congestion pricing via network congestion game approach," *Discrete and Continuous Dynamical Systems - S*, vol. 14, 01 2018.
- [2] P. N. Brown and J. R. Marden, "Can taxes improve congestion on all networks?" *IEEE Transactions on Control of Network Systems*, vol. 7, no. 4, pp. 1643–1653, 2020.
- [3] L. Zhang, K. Gong, and M. Xu, "Congestion control in charging stations allocation with Q-learning," *Sustainability*, vol. 11, no. 14, 2019.
- [4] D. Paccagnan, M. Kamgarpour, and J. Lygeros, "On aggregative and mean field games with applications to electricity markets," in *2016 European Control Conference (ECC)*, 2016, pp. 196–201.
- [5] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros, "Distributed computation of generalized Nash equilibria in quadratic aggregative games with affine coupling constraints," in *2016 IEEE 55th Conference on Decision and Control (CDC)*, 2016, pp. 6123–6128.
- [6] —, "Nash and Wardrop equilibria in aggregative games with coupling constraints," *IEEE Transactions on Automatic Control*, vol. 64, no. 4, pp. 1373–1388, 2019.
- [7] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 67–78, 2013.
- [8] W. Tushar, W. Saad, H. V. Poor, and D. B. Smith, "Economics of electric vehicle charging: A game theoretic approach," *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1767–1778, 2012.
- [9] A. Laha, B. Yin, Y. Cheng, L. X. Cai, and Y. Wang, "Game theory based charging solution for networked electric vehicles: A location-aware approach," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 7, pp. 6352–6364, 2019.
- [10] N. Groot, B. De Schutter, and H. Hellendoorn, "Reverse Stackelberg games, Part I: Basic framework," *2012 IEEE International Conference on Control Applications*, pp. 421–426, 2012.
- [11] K. Staňková, G. Olsder, and M. Bliemer, *Bi-level optimal toll design problem solved by the inverse Stackelberg games approach*, 01 2011, vol. 89, pp. 871–880.
- [12] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave  $N$ -person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.
- [13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, March 2004.
- [14] D. Monderer and L. Shapley, "Potential games," *Games and Economic Behavior*, vol. 14, pp. 124–143, 05 1996.
- [15] S. Grammatico, F. Parise, M. Colombino, and J. Lygeros, "Decentralized convergence to Nash equilibria in constrained deterministic mean field control," *IEEE Transactions on Automatic Control*, vol. 61, no. 11, p. 3315–3329, Nov 2016.
- [16] V. Berinde, "Iterative approximation of fixed points," *Lecture Notes in Mathematics*, vol. 1912, 01 2007.
- [17] P. Harker and J.-S. Pang, "Finite-dimensional variational inequality and nonlinear complementarity problems: A survey of theory, algorithms and applications," *Math. Program.*, vol. 48, pp. 161–220, 03 1990.