A macroscopic modelling framework for the dynamic pricing of pool ride-splitting vehicles in bus lanes

Lynn Fayed[1] and Gustav Nilsson[1] and Nikolas Geroliminis[1]

Abstract— **The impact of ride-hailing vehicles on congestion raises multiple concerns, particularly in areas where the network space is constrained and the route infrastructure is unevenly distributed among multi-modal users. Mainly, idle ride-hailing vehicles pose multiple challenges because they move across the network and contribute to production without delivering any trips. One possible solution to halt the negative effects of ride-hailing on traffic in a network is trip-sharing. To incentivize users to share their rides, we provide in this work a macroscopic dynamic framework for multi-modal networks with on-demand ride-hailing services where pool passengers are allowed on bus lanes. We then develop a pricing policy for solo and pool trips to reduce overall multi-modal network delays. Using a PI and a Model Predictive Control (MPC) framework, we regulate the price difference between the two ride-hailing alternatives with the objective to minimize the Passenger Hours Travelled (PHT) for bus users but also for the users of other concurrent transportation modes. The results show that the ideal set point for the PI controller is heavily dependent on the level of demand. The MPC framework, despite being more complex from an implementation point of view, manages to return lower total network delays.**

I. Introduction

The ubiquitous character of ride-hailing is attributed to the convenient and flexible door-to-door service it provides to users at very affordable rates. However, its success and freedom of operation were soon hindered by the attempts of network regulators to curtail its influence on traffic and to mitigate its impact on public transport usage. In fact, the high number of idle vehicles improves ride-hailing's level of service, but it also increases the number of empty Vehicle Kilometers Travelled (VKT) and congestion levels [1]. Moreover, ride-hailing is sometimes labeled as a public transport competitor because it presents itself as a substitutionary service [2]. This is particularly the case in areas where public transit is deficient, unreliable, and not well-connected with the main network hubs.

Ride-splitting is one possible solution with the potential to counteract some of the negative externalities of ride-hailing. By pooling passengers in a single trip, platforms are able to reduce their total VKT [3] and their fleet size [4], while guaranteeing the same service level. Because ride-splitting is mostly efficient when the engagement level in pooling is high [5], an incentive-based upfront fare discount is generally offered to users if they consent to sharing their rides. Its main purpose is to compensate for the additional travel time that sharing users may incur due to the pick-up and drop-off of other users.

Proposing efficient and well-aimed regulatory strategies requires, first and foremost, a proper understanding of the structure and operation of ride-hailing markets. In [6], the authors present an aggregate equilibrium modelling framework for ride-hailing and highlight the difference in market states between an efficient service where vehicles are mostly idling and an inefficient service where idle vehicles are sent far away to pick up passengers. A similar comprehensive economic model is presented in [7] where service pricing and fleet sizing are determined under profit and social welfare maximization scenarios. In [8], the authors assessed the aggregate equilibrium in ride-splitting markets, and in contrast with [9], they integrated in their framework a macroscopic traffic model to capture the impact of ride-hailing vehicles on congestion. They then used the model they developed to demonstrate the power of enforcing a platform commission cap and a congestion toll to achieve sustainable equilibrium solutions. Another regulatory approach is examined in [10], where the authors propose an occupancy-dependent space allocation strategy that improves multi-modal exploitation of the limited network space. Again, they resorted to an aggregate and static network equilibrium model to show that allowing pool ride-hailing trips in bus lanes could improve overall network delays. However, they also argued that without any pricing strategies, this benefit could possibly be reversed due to additional delays caused to bus users by pool vehicles.

The contribution of this paper is twofold. First, we develop a dynamic macroscopic model for multi-modal networks with private vehicles, ride-hailing services, and buses. This model is built according to an occupancy-dependent network space allocation policy where private vehicles and solo ride-hailing users travel in one portion of the network whereas bus riders and pool ride-hailing users travel in the remaining network portion. Second, we utilize this model to advance a control framework for differential pricing of solo and pool ride-hailing alternatives to steer the overall network towards its system optimum. This suggested network regulatory policy therefore aims at minimizing the total Passenger Hours Travelled (PHT) for multi-modal users by encouraging or deterring ride-hailing users from pooling in bus lanes. We investigate two different control strategies, one myopic Proportional-Integral (PI) control strategy and one Model Predictive Control (MPC) strategy. We also assess the control strategies' robustness with respect to abandonment, i.e., when

[1]The authors are with the Urban Transport Systems Laboratory (LUTS), École Polytechnique Fédérale de Lausanne (EPFL), Switzerland. {lynn.fayed,gustav.nilsson, nikolas.geroliminis}@epfl.ch

This work was supported by the Swiss National Science Foundation under NCCR Automation, grant agreement 51NF40_180545.

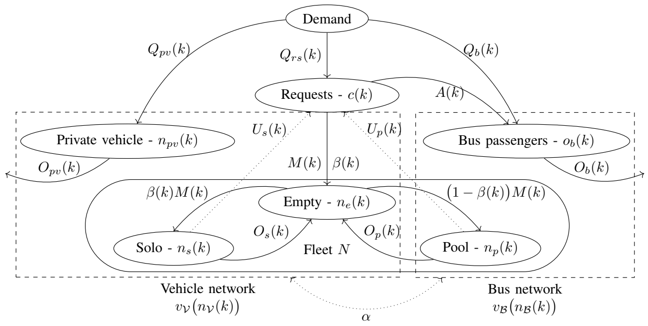

Fig. 1. A schematic sketch of the suggested multi-modal space allocation framework we assess in this paper

the waiting time for a ride-hailing trip becomes too long and the users choose another travel mode instead.

The remainder of the paper is outlined as follows. In Section II, we set forth the macroscopic occupancy-dependent and modal-based space allocation strategy, and we accordingly define the network dynamics. Next in Section III, we elaborate on the control frameworks that we implement to steer the network towards a system optimum through ride-hailing service pricing, and we present the results in the numerical example section, Section IV. Finally, the paper concludes with the main findings and proposes elements for future research in Section V.

II. Model

A. Space allocation framework

Consider a multi-modal network with a set of available mode alternatives $\mathcal{M}$: private vehicles pv, buses b, or ride-splitting services rs, such that $\mathcal{M} := \{pv, b, rs\}$. We will model the dynamics in discrete time with time step duration $\tau > 0$ and $k \in \mathcal{K} := \{0, \ldots, k_{\max}\}$ denoting the time step and $\bar{\mathcal{K}} := \mathcal{K} \backslash \{k_{\max}\}$. The modal and time-dependent demand expressed in passengers per hour is exogenous, and is given by $Q_j(k)$ for $j \in \mathcal{M}$. Ride-hailing users have the choice to travel solo or to share their rides with exactly one other passenger for a fraction of their trips. We refer to these two trips by s and p, respectively, and we denote by $\beta(k) \in [0, 1]$ the fraction of ride-hailing users opting for a solo trip at time step $k \in \mathcal{K}$.

We split the network under consideration into a vehicle network $\mathcal{V}$ occupying a fraction $\alpha \in [0, 1]$ of the total available space, and a bus network $\mathcal{B}$ spanning over a fraction $\bar{\alpha} = 1 - \alpha$. In our model, private vehicles utilize the vehicle network $\mathcal{V}$ and bus users utilize the bus network $\mathcal{B}$ at all times. For ride-hailing trips, users opting for a solo ride travel in the vehicle network $\mathcal{V}$ whereas users choosing to pool travel in the high occupancy bus network $\mathcal{B}$.

The total ride-hailing fleet size $N > 0$ is fixed, and its vehicles at any point in time $k \in \mathcal{K}$ can be in either of the following states: empty, assigned to a solo trip and occupied by one passenger, or assigned to a pool trip and occupied by at least one passenger and a maximum of two passengers. The number of ride-hailing vehicles in each state, which we model as a continuous quantity, is $n_e(k)$, $n_s(k)$, and $n_p(k)$, respectively, such that $N = n_e(k) + n_s(k) + n_p(k)$ for all $k \in \mathcal{K}$. Similarly, we denote by $n_{pv}(k)$ the time-dependent number of private vehicles and by n_b the number of buses here assumed to be fixed. Accordingly, the accumulation in the vehicle network $n_{\mathcal{V}}$ at any point in time $k \in \mathcal{K}$ is given by $n_{\mathcal{V}}(k) = n_{pv}(k) + n_e(k) + n_s(k)$, and the accumulation in the bus network $n_{\mathcal{B}}$ at any point in time $k \in \mathcal{K}$ is $n_{\mathcal{B}}(k) = n_p(k) + n_b$. Notice here that empty ride-hailing vehicles are only allowed to travel in the vehicle network $\mathcal{V}$.

B. Aggregate traffic model

In the following part, we elaborate on the macroscopic traffic dynamic model proposed to identify the relationships between network production P, network accumulation n, and network speed v. The total production $P : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a function of n such that $P\big(n(k)\big) = n(k)v\big(n(k)\big)$ where $v : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is assumed to be decreasing with the accumulation such that $\frac{\mathrm{d}v}{\mathrm{d}n} \leq 0$. Accordingly, we compute the production in $\mathcal{V}$, $P_{\mathcal{V}} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, and the production in $\mathcal{B}$, $P_{\mathcal{B}} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, using the space allocation factor α such that $P_{\mathcal{V}}(\alpha n(k)) = \alpha P(n(k))$ and $P_{\mathcal{B}}(\bar{\alpha} n(k)) = \bar{\alpha} P(n(k))$ respectively [11], [12]. Similarly, the speed in the vehicle network $v_{\mathcal{V}}$ and in the bus network $v_{\mathcal{B}}$ are given by $v_{\mathcal{V}}\big(\alpha n(k)\big) = v\big(n(k)\big)$ and $v_{\mathcal{B}}\big(\bar{\alpha} n(k)\big) = v\big(n(k)\big)$. Rewriting the production functions in terms of speed, we obtain that $P_{\mathcal{V}}(n_{\mathcal{V}}(k)) = n_{\mathcal{V}}(k)v_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)$ and $P_{\mathcal{B}}(n_{\mathcal{B}}(k)) = n_{\mathcal{B}}(k)v_{\mathcal{B}}\big(n_{\mathcal{B}}(k)\big)$. Furthermore, given that the marginal impact of a bus and a pool vehicle on traffic in $\mathcal{B}$ is not equivalent, we partition the production in the bus network into pool vehicle production $P_p : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and bus production $P_b : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, each dependent on both values of n_p and n_b. Buses need to repetitively board and alight passengers at stops, and we capture this action by reducing $v_{\mathcal{B}}$ with a factor $r(n_b)$, where $r : \mathbb{R}_{\geq 0} \to (0, 1]$ and $\frac{\mathrm{d}r}{\mathrm{d}n_b} < 0$. The running speed of the pool vehicles in $\mathcal{B}$ is given by $v_p\big(n_p(k), n_b\big) = v_{\mathcal{B}}\big(n_{\mathcal{B}}(k)\big)r(n_b)$ and the operational speed of buses in $\mathcal{B}$, including the time they spend at stops, is

$$v_b\big(n_p(k), n_b\big) = \frac{v_p\big(n_p(k), n_b\big)}{1 + v_p\big(n_p(k), n_b\big)\frac{\bar{t}_d}{\bar{s}}},$$

where $\bar{t}_d$ and $\bar{s}$ are the average dwell time of buses and the spacing between stops, respectively. Therefore, pool and bus production functions become $P_p\big(n_p(k), n_b\big) = n_p(k)v_p\big(n_p(k), n_b\big)$ and $P_b\big(n_p(k), n_b\big) = n_b v_b\big(n_p(k), n_b\big)$ respectively for all $k \in \mathcal{K}$.

C. System dynamics

Previously, we put forward an occupancy-dependent allocation scheme and defined an aggregate traffic model for the network under consideration. Accordingly, we proceed with finding the state dynamics for the different modes under consideration.

First, let $O_{pv}(k)$ denote the number of private vehicles that are completing their trips and leaving the network at time step k. Assuming a constant average trip distance of $\bar{l}_{pv} > 0$ and a homogeneous mixture of private and ride-hailing vehicles, $O_{pv}(k)$ is given by $O_{pv}(k) = \frac{n_{pv}(k)}{n_{\mathcal{V}}(k)} \frac{P_{\mathcal{V}}(n_{\mathcal{V}}(k))}{l_{pv}}$. The accumulation of private vehicles between any two time steps is modelled with a discrete dynamic formulation as

$$\Delta n_{pv}(k) = \tau \left[\frac{Q_{pv}(k)}{\bar{o}_{pv}} - O_{pv}(k) \right], \forall k \in \bar{\mathcal{K}}, \quad (1)$$

where $\bar{o}_{pv} > 0$ is the average occupancy of a private vehicle. Moreover, we let Δ denote the forward difference between two consecutive time steps, i.e., $\Delta n_{pv}(k) = n_{pv}(k+1) - n_{pv}(k)$.

The dynamics of the ride-splitting services are more complex as they involve different types of vehicle categories utilizing separate networks. Requests arrive to the platform at every time step and are given the choice to pool or not according to the prevailing conditions in every network. In practice, this choice is dependent not only on the solo fare $\tilde{F}_s(k)$ and pool fare $\tilde{F}_p(k)$, but also on the expected travel time of every individual option. Let $U_s(k)$ and $U_p(k)$ denote the disutilities for a solo or a pool trip, respectively, at time step k, then their expressions are given by $U_s(k) = \tilde{F}_s(k) + \kappa \frac{\bar{l}_s}{v_{\mathcal{V}}(n_{\mathcal{V}}(k))}$ and $U_p(k) = \tilde{F}_p(k) + \kappa \frac{\bar{l}_s + l_p^{\Delta}}{v_p(n_p(k), n_b)}$, where $\kappa > 0$ is the value of time, $\bar{l}_s > 0$ is the average trip length for a solo trip, and $l_p^{\Delta} > 0$ is the pool detour distance that passengers incur in case they opt for pooling. In this work, we consider that $\tilde{F}_s(k)$ is constant such that $\tilde{F}_s(k) = F_s$ for all $k \in \mathcal{K}$, F_s being the solo trip fare set by the ride-hailing platform. On the contrary, the expression for $\tilde{F}_p$ is given by $F_p + \phi(k)$ where F_p is the platform static pool trip fare and $\phi(k) \in \mathbb{R}$ is the control fare that steers the system towards a predetermined objective that we elaborate on in Section III. In accordance with the spatial strategy proposed, solo passengers travel in network $\mathcal{V}$ with a speed $v_{\mathcal{V}}$ and pool passengers travel in $\mathcal{B}$ with a speed v_p. For ease of future implementation, let $u_s(k)$ and $u_p(k)$ be the solo and pool trip disutilities if no intervention is expected, then $u_s(k) = U_s(k)$ and $u_p(k) = U_p(k) - \phi(k)$. Using a binary logit model, the fraction of ride-hailing passengers that opts for the solo option $\beta(k)$ out of the total ride-hailing demand at k is given by

$$\beta(k) = \frac{\exp\big(-\mu u_s(k)\big)}{\exp\big(-\mu u_s(k)\big) + \underbrace{\exp\left(-\mu \phi(k)\right)}_{\xi(k)} \exp\big(-\mu u_p(k)\big)},$$

where $\mu > 0$ is the model scale parameter and $\xi(k) \in (0, +\infty)$ is our control variable. Once $\xi(k)$ is determined, the prices can be computed as $\phi(k) = \frac{\log(\xi(k))}{-\mu}$. It follows that, if $c(k)$ represents the total waiting requests at time step k, then the number of requests opting for a solo trip is $c_s(k) = \beta(k) c(k)$, and the number of those opting for a pool trip is $c_p(k) = \big(1 - \beta(k)\big) c(k)$. With respect to the matching technology between empty ride-hailing vehicles n_e and waiting requests c, we adopt a bilateral meeting function using a Cobb-Douglas formulation extended to a time-dependent framework [13]. The matching rate between vehicles and requests at time step $k \in \mathcal{K}$ is therefore approximated by

$$M(k) = a_0 n_e(k)^{\alpha_e} \left(c_s(k) + \frac{1}{2} c_p(k) \right)^{\alpha_c},$$

where $a_0 > 0$, $\alpha_e > 0$, and $\alpha_c > 0$ are the Cobb-Douglas meeting function parameters. We note here that the number of requests waiting to be matched with an empty vehicle at time step k is equal to $c_s(k) + \frac{1}{2} c_p(k)$ as only half of the requests opting for pooling are picked up by idling vehicles. If the matching and pooling decisions are determined at the end of every time step, and the idle vehicle dispatching occurs at the start of the subsequent time step, then the change in the number of idling vehicles between two consecutive time steps k and $k+1$ is

$$\Delta n_e(k) = \tau \left[\frac{n_s(k)}{n_{\mathcal{V}}(k)} \frac{P_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)}{\bar{l}_s} + \frac{P_p\big(n_p(k), n_b\big)}{\bar{l}_s + l_d^{\Delta}} - M(k) \right], \quad (2)$$

for all $k \in \bar{\mathcal{K}}$. The first two elements of (2) represents the inflow into the category of empty vehicles, i.e., the rate of trip completion for solo and pool vehicles that we denote by $O_s(k)$ and $O_p(k)$ respectively. The former is dependent on the production in $\mathcal{V}$ and the average trip distance of a solo trip $\bar{l}_s > 0$. The latter is defined using the vehicle production in the bus network P_b but also the total pool trip distance $\bar{l}_s + l_d^{\Delta}$ where $l_d^{\Delta} > 0$ is the pool driver detour. The third element of (2) is the rate of vehicles flowing out of the empty category n_e to join the solo or pool vehicle categories n_s and n_p. Therefore, the discretized dynamics for solo vehicles are

$$\Delta n_s(k) = \tau \left[\beta(k) M(k) - \frac{n_s(k)}{n_{\mathcal{V}}(k)} \frac{P_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)}{\bar{l}_s} \right], \forall k \in \bar{\mathcal{K}}, \quad (3)$$

and the discretized dynamics for pool vehicles are

$$\Delta n_p(k) = \tau \left[\bar{\beta}(k) M(k) - \frac{P_p\big(n_p(k), n_b\big)}{\bar{l}_s + l_d^{\Delta}} \right], \forall k \in \bar{\mathcal{K}}. \quad (4)$$

The rate of vehicle-request matches that enter the solo vehicle category n_s at time step $k+1$ is $\beta(k) M(k)$ and the matches that enter the pool vehicle category is $\bar{\beta}(k) M(k)$ where $\bar{\beta}(k) = 1 - \beta(k)$. Analogously, the changes in the number of ride-hailing customers in the queue with time are given by $c(k+1) = c(k) + \tau[Q_{rs}(k) + \big(\beta(k) - 2\big) M(k)] - A(k)$ for all $k \in \mathcal{K}$, where $Q_{rs}(k)$ is the ride-hailing demand at k and $A(k)$ is the number of abandoning requests due to long waiting times. Therefore, if the passenger waiting tolerance is $w_{\max} > 0$, then we estimate $A(k) = \max\left(c(k) - \frac{1}{k} \sum_{\tilde{k}=1}^{k} M(\tilde{k}) w_{\max}, 0\right)$, which represents the number of passengers leaving the waiting queue. In our framework, abandoning ride-hailing requests board the buses instead.

Finally, given that the number of buses n_b in the network $\mathcal{B}$ is constant, the bus dynamics are therefore narrowed down to tracking eassenger occupancy per bus o_b. Therefore, the discretized changes in o_b between two consecutive time steps k and $k+1$ are

$$\Delta o_b(k) = \frac{\tau}{n_b} \left[Q_b(k) + A(k) - \frac{P_b\big(n_p(k), n_b\big)}{\bar{l}_b} o_b(k) \right], \quad (5)$$

for all $k \in \bar{\mathcal{K}}$, where $\bar{l}_b$ is the average trip length by bus. The inflow in (5) is the bus demand Q_b and the abandoning ride-hailing passengers A. The last term in (5) represents the outflow O_b which is the passenger trip completion rate. Note that all these elements are divided by n_b to compute the average occupancy per bus, which is assumed to be uniform among all operating buses.

Figure 1 provides a summary sketch of the space allocation strategy put forward in this work as well as the dynamics for the different transportation modes utilizing the network infrastructure.

III. Control Strategies

Theoretically, the goal of allowing pool ride-hailing vehicles in bus lanes is to improve the overall traffic conditions in multi-modal networks without significantly impacting bus flow. In the following section, we elaborate on the PI control and MPC framework that we develop to achieve this goal.

A. PI control

The space allocation strategy we suggest interferes with bus flow, and this interference must be contained within acceptable levels to maintain a good public transport service level. We achieve this objective by developing a PI controller which goal is to minimize the error term, here given by the difference between the target and actual bus speeds $\bar{v}_b$ and $v_b(n_p(k), n_b)$, respectively. Therefore, if $\epsilon(k) = \big(\bar{v}_b - v_b(n_p(k), n_b)\big)$, then the expression for the control variable ϕ at $k \in \mathcal{K}$ is

$$\phi(k) = K_p \epsilon(k) + \frac{K_i}{N_e} \sum_{\tilde{k}=\max(k-(N_e+1),0)}^{k-1} \epsilon(\tilde{k}) \,,$$

where $K_p > 0$ and $K_i \geq 0$ are the constants for the proportional and integral terms of the PI controller. Notice that for the integral term, we keep track of the previous errors for the last $N_e \in \mathbb{N}$ time steps.

B. Model Predictive Control

Unlike the PI controller which exclusively focuses on bus delays, the MPC framework minimizes delays for the different network users, including private vehicles, solo and pool ride-hailing users, and bus passengers. Consequently, if the total Passenger Hours Travelled (PHT) by multi-modal users at time step $k \in \mathcal{K}$ is $\mathrm{PHT}(k) = \tau[n_{pv}(k)\bar{o}_{pv} + n_b o_b(k) + n_s(k) + n_p(k)\bar{o}_p]$, the formulation of the MPC framework is given by

$$\begin{aligned} \text{minimize} \quad & \sum_{k\in\mathcal{K}} \mathrm{PHT}(k) \\ \text{subject to} \quad & \xi(k) \in [\xi_{\min}, \xi_{\max}] \quad \forall k \in \mathcal{K} \\ & \xi(k) = \xi(k-1) \quad \forall k \in \mathcal{K} \setminus \{n \cdot N_u \mid n \in \mathbb{N}\} \\ & |\xi(k) - \xi(k-1)| \leq \sigma \quad \forall k \in \mathcal{K} \\ & (1), (2), (3), (4), (5) \end{aligned}$$

where $\xi_{\min}$ and $\xi_{\max}$ are the exogenous lower and upper bounds for the control variable, and $\bar{o}_p \in (1, 2]$ is the average occupancy of a pool trip. We constrain the control action to only be updated every $N_u \in \mathbb{N}$ time steps. Moreover, we make sure that the difference between any two update steps does not exceed the exogenous limit σ.

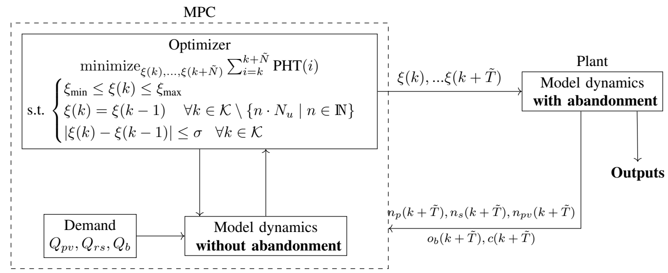

Fig. 2. Implementation of the MPC framework with abandonment

TABLE I
Macro-simulation parameters

Variable description	Variable name	Value	Unit
Trip length for private vehicles	l_{pv}	3.86	km
Trip length for solo rides	$\bar{l}_s$	3.86	km
Trip length for buses	$\bar{l}_b$	5.4	km
Driver pool trip detour	l_d^{Δ}	2.7	km
Passenger pool trip detour	l_p^{Δ}	0.6	km
Private vehicle occupancy	$\bar{o}_{pv}$	1.2	pax
Pool occupancy	$\bar{o}_p$	1.5	pax
Scale parameter	μ	1	-
Value of time	κ	30	CHF/hr
Fare for solo trip	F_s	5	CHF
Fare for pool trop	F_p	4	CHF
Fleet size	N	3500	veh
Number of buses	n_b	530	bus
Length of time step k	τ	6	s

In the following section, we elaborate on the MPC framework using a numerical example. We furthermore compare the multi-modal user delays for the no control, PI control, and MPC implementations.

IV. Numerical Study

For demonstration purposes, we consider a network with a total production $P(n) = A_0 n^3 + B_0 n^2 + C_0 n$, such that $A_0 = 5.74 \cdot 10^{-9}$, $B_0 = -1.02 \cdot 10^{-3}$, and $C_0 = 36$ for $n \in [0, 58536]$. The vehicle network $\mathcal{V}$ occupies a fraction $\alpha = 0.8$ of the infrastructure space, and the remaining fraction is allocated to the bus network $\mathcal{B}$. This fraction α is crucial in our framework because it yields an expression for both $P_{\mathcal{V}}$ and $P_{\mathcal{B}}$. To be able to compute P_p from $P_{\mathcal{B}}$, we capture the marginal influence of buses on other vehicles by reducing the bus network speed using the function $r(n_b) = e^{-6.5 \cdot 10^{-4} n_b}$. Finally, P_b is computed by setting the spacing between bus stations $\bar{s}$ to 0.8 km and the boarding and alighting time of passengers $\bar{t}_d$ to 30 s. The matching function takes the values 0.025, 0.93, and 0.98 for a_0, α_e, and α_c respectively. The remaining constant parameters used in our macroscopic simulation are listed in Table I.

The implementation of the PI controller requires a proper selection of the speed set point $\bar{v}_b$. For this reason, we use the equilibrium steady-state model with fixed demand presented in [10] and combine it with a binary logit mode choice to plot in Figure 3 the optimal operational set points. In our framework, the optimal set point is the bus speed

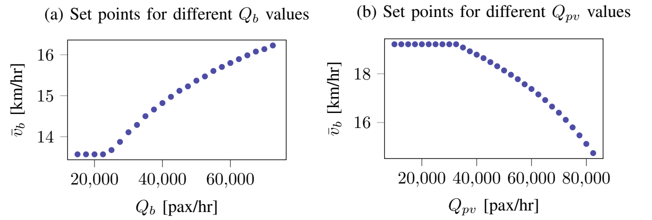

Fig. 3. Variation of $\bar{v}_b$ for different bus and private vehicle demand rates

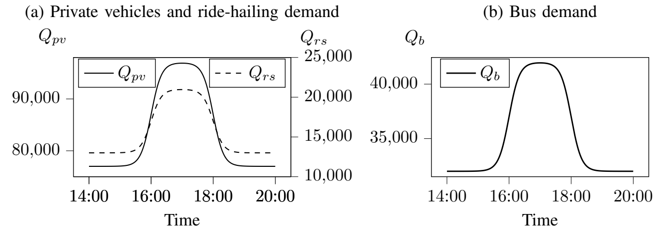

Fig. 4. Time-dependent multi-modal demand profile

for which the multi-modal user delay is minimized. When $Q_{pv} = 84000$ pax/hr and $Q_{rs} = 15000$ pax/hr, Figure 3(a) shows that the optimal bus speed set point increases with the bus demand, indicating that our allocation strategy becomes more critical when the bus demand is high. The same logic applies in Figure 3(b) for $Q_b = 35000$ pax/hr and variable private vehicle demands, where we observe that the optimal operational bus speed set point decreases with private vehicle demand. This indicates that a lower operational point for buses is acceptable in network $\mathcal{B}$ when the private vehicle demand is very high in network $\mathcal{V}$. The previous analysis is performed to provide intuition on the choice of the bus speed set point for the PI controller in the dynamic multi-modal macro-simulation.

Next, we simulate the network dynamics using the demand profiles displayed in Figure 4(a) for private vehicles and ride-hailing users, and in Figure 4(b) for bus users. The simulation spans over a duration of 6 hours, reproducing the evening peak in between two off-peak periods. We start with the case where w_{max} is set to infinity, meaning that solo or pool ride-hailing users do not abandon the system to travel by bus. Table II shows the total user delays for scenarios where (i) all ride-hailing users travel in $\mathcal{V}$, (ii) all ride-hailing users travel in $\mathcal{B}$, (iii) solo users travel in $\mathcal{V}$ and pool users travel in $\mathcal{V}$ with no intervention, (iv) a PI control implementation, and (v) an MPC implementation. Moreover, the set point for the PI controller in scenario (iv) is chosen by taking the average of the dynamic bus demand in Figure 4(b) and selecting the optimal set point for this value of average bus demand from the graph in Figure 3(a). With respect to the MPC parameter settings, we fix ξ_{min} and ξ_{max} to $5 \cdot 10^{-5}$ and 30 respectively, N_u to 180 time steps, and σ to 7. While scenario (i) is infeasible and scenario (ii) yields relatively high delays, the proposed allocation strategy has the potential to reduce multi-modal delays even without any regulatory price intervention. The PI control implementation performs worse than the no control scenario, and this is mainly attributed to the choice of set points. In fact, the PI controller only accounts for bus speed, and even with a proper choice of $\bar{v}_b$, scenario (iv) does not guarantee low PHT values because the total multi-modal user delay is also dependent on the private vehicle and ride-hailing demand. Finally, the MPC returns solutions with the lowest delays, mainly because its objective function considers all mode users.

When accounting for ride-hailing requests' impatience within our framework, requests are expected to abandon the platform. Accordingly, we set $w_{\text{max}} = 15$ min and report the results for the same scenarios in Table II. The results are consistent with what we observed with the macro-simulation not taking abandonment into account, with the MPC scenario yielding the lowest delays. We note here that the MPC implementation is not straightforward for scenarios with abandonment. Therefore, to overcome this complexity, we follow the approach presented in Figure 2 where we include abandonment in the plant dynamics but exclude it from the MPC framework. The prediction horizon for the optimizer is set to $\tilde{N} = 3600$ time steps, and we update our state variable according to the actual plant dynamics using the output control variables every $\tilde{T} = 600$ time steps.

TABLE II

PHT FOR THE SCENARIOS WITHOUT AND WITH ABANDONMENT

Controller	**PHT [pax.km/hr]**	**PHT [pax.km/hr]**	**Abandonment**
All Q_{rs} in $\mathcal{V}$	Infeasible	Infeasible	Infeasible
All Q_{rs} in $\mathcal{B}$	198473	202057	14133
No control	193019	194184	6110
PI	195141	196805	8242
MPC	190601	192480	7998

The state and control variables for the PI and MPC implementations with abandonment are presented in Figures 5 and 6 respectively. Clearly, as the PI and the MPC achieve separate objectives, the variations of the state variables are not the same. For a set point $\bar{v}_b$ equal to 14.5 km/hr (Figure 5(d)), when the bus network capacity allows it, the PI controller provides a higher pool discount ϕ as shown in Figure 5(g), therefore sending more pool vehicles to the bus network. This is concluded by mainly comparing the number of solo and pool vehicles in Figure 5(a) and Figure 5(b) respectively. We mention here that when the intergral gain value K_i is set to 30 and N_e to 500, the controller oscillates more but is capable of fluctuating around the set point in Figure 5(d) compared to the case of P control only with $K_i = 0$, where v_b does not fully get to $\bar{v}_b$. Moving to the MPC framework, it is clear from Figures 6(a) and 6(b) that pooling is incentivized during the peak period, and this is explained by the negative ϕ values in Figure 6(g) offered to pool passengers to encourage them to travel in bus lanes. Note that the control variable ϕ sometimes takes positive values, indicating that pool ride-hailing users have to pay extra to utilize the bus lanes, as conditions in the bus network $\mathcal{B}$ are better than the vehicle network $\mathcal{V}$.

V. CONCLUSION

In this work, we provide a macroscopic dynamic model for a multi-modal space allocation strategy that allows pool

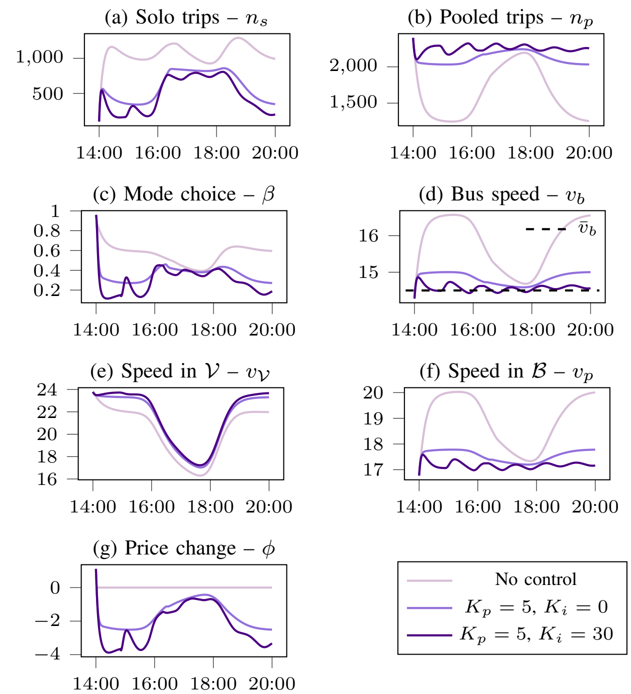

Fig. 5. Time-dependent model variables for the PI controller with abandonment

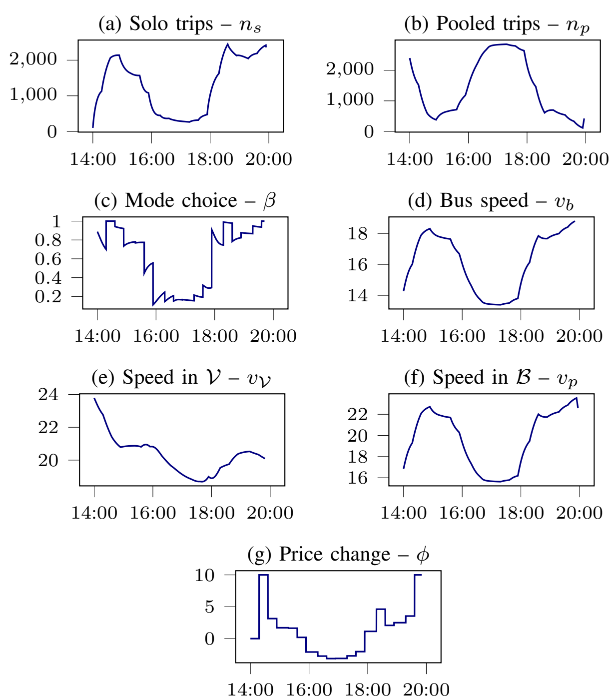

Fig. 6. Time-dependent model variables for the MPC with abandonment

ride-hailing users in bus lanes. We then utilize the model to develop two control frameworks to guarantee the improvement of total network delays, and to avoid the amplification of bus users' travel time. First, the PI controller aims at incentivizing or discouraging pooling in bus lanes by setting a pricing scheme that minimizes the difference between the target and actual bus speeds. We show that this implementation is highly demand-dependent, and the choice of set points does not guarantee good solutions for all multi-modal users. Second, the MPC framework sets the price for pooling in bus lanes with the aim of minimizing the total PHT of all mode users in the network. Whether we account for abandonment in our framework or not, the MPC has the potential to always result in a pricing scheme that is optimal on the network level. Future work will mainly consider the possibility of adding a new ride-hailing choice alternative by giving pool users the option to either travel in the vehicle or bus network with the aim of determining the price discrimination between the different alternatives accordingly.

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